MARCO MALANDRONE

Let's ignore ~~over~~ ~~each~~ all collisions while B is changing, due to the fact that $t \gg \tau$.

At first each electron has an almost equally distributed ~~speed~~ velocity along the three axis. Let's suppose the z-axis is the one with the same direction of $\vec{B}$.

DEFINITION: $v_\perp^2 = v_x^2 + v_y^2$

Since $\tau \gg m/(Be)$ each electron is going to have a nearly-circular trajectory (on the $z=k$ plane) with radius:

$$R = \frac{v_\perp m}{eB}$$

Using the Faraday-Newmann-Lenz law:

$$-\pi R^2 \frac{dB}{dt} = f_{em} \Rightarrow$$

$$\Rightarrow -\pi R^2 \frac{dB}{dt} \cdot e \cdot dt = f_{em} \cdot e \cdot dt \Rightarrow$$

$$\Rightarrow \pi R^2 \frac{dB}{dt} \cdot e \cdot dt = -F \cdot 2\pi R \cdot dt = -2\pi R \cdot dI =$$

$$= -2\pi R \cdot m \cdot dv_\perp \Rightarrow$$

$$\Rightarrow R e \, dB = -2m \, dv_\perp \Rightarrow \frac{v_\perp m}{eB} e \, dB = -2m \, dv_\perp \Rightarrow$$

$$\Rightarrow -\frac{1}{B} dB = 2 \cdot \frac{1}{v_\perp} dv_\perp \Rightarrow$$

$$\Rightarrow -\ln\left(\frac{2B_0}{B_0}\right) = \ln\left(\left(\frac{v_{\perp 1}}{v_{\perp 0}}\right)^2\right)$$

where $v_{\perp 1}$ is $v_\perp$ when $B = 2B_0$

$v_{\perp 0}$ is $v_\perp$ at first when $B = B_0$

Then $v_{\perp 1}^2 = \frac{1}{2} v_{\perp 0}^2$

Initially $E_0 = \frac{1}{2} m \left(v_{z0}^2 + v_{x0}^2 + v_{y0}^2 \right) = \frac{1}{2} m \left(v_z^2 + v_{\perp 0}^2 \right)$

When $B = 2B_0$ $\quad E_1 = \frac{1}{2} m \left(v_z^2 + v_{\perp 1}^2 \right) = \frac{1}{2} m \left(v_z^2 + \frac{1}{2} v_{\perp 0}^2 \right) = \frac{2}{3} E_0$

We must now redistribute the energy along the three axis:

$$v_z'^2 = \frac{1}{3} \frac{2E_1}{m} = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{E_0}{m} = \frac{4}{9} \frac{E_0}{m}$$

$$v_{\perp 1}'^2 = \frac{8}{9} \frac{E_0}{m}$$

We will have $v_{\perp 2}^2 = 2 v_{\perp 1}'^2$ when B will be back to B_0

$$E_2 = \frac{1}{2} m \left(v_z'^2 + v_{\perp 2}^2 \right) = \frac{1}{2} m \left(\frac{4}{9} \frac{E_0}{m} + \frac{16}{9} \frac{E_0}{m} \right) = \frac{10}{9} E_0$$

Since $\frac{E_2}{E_0} = \frac{T'}{T_0} \Rightarrow T' = T_0 \cdot \frac{E_2}{E_0} = T_0 \cdot \frac{10}{9}$

$$T' = T_0 \cdot \frac{10}{9}$$

MARCO MALANDRONE

marco.malandrone@gmail.com