Firstly we find the center of the ellipse. To find it, we simply draw two pairs of parallel chords of the ellipse, and intersect the lines connecting their midpoints. This is easy to see because the stretch that sends this ellipse to its auxiliary circle preserves midpoints and the center of the conic. Now we draw the line OP, where O is the center of the ellipse and P is the image of the center of the circle. Let it intersect the ellipse at A, A'. Choose any one of them, say A. Draw 4 random points on the ellipse say B, C, D, E. From Pascal's theorem on the degenerate hexagon $AABCDE$, we get that the points $AA \cap CD$, $AB \cap DE$, $AE \cap BC$ are collinear. So to construct the tangent to the ellipse we join $AB \cap CD$, $AE \cap BC$ and let it intersect with CD. The intersection point when joined to A gives the tangent at A. Draw a line parallel to this tangent through P. This line is bisected by P, as can be seen again by the stretch transform we mentioned. Let this line meet the ellipse at F, G. Suppose F', G' were the points on the circle which had been diametrically opposite, and the line joining them was perpendicular to the principal axis. Then since it is known that the image of $F'G'$ is a line perpendicular to the principal axis. Also since the midpoint of FG maps to the midpoint of the images of FG (due to a simple homothety at the optical center), so we know that F, F and G, G are conjugate with respect to the lens, since there is a unique chord through a non-center point that is bisected by that point. This again can be seen from the stretch. Now note that tangents to the circle become tangents to the conic, due to continuity of the coordinates of the point. So the tangents to the ellipse at F, G meet at the point which was the image of the point at infinity along the principal axis, which is precisely the image focus. To find the focal plane, we need to find the intersections $AF \cap AG$ and $AF \cap AG$. They are the images of two points on the line at infinity, and thus comprise the focal plane. So we get the principal axis as well, the line through the secondary focus and perpendicular to the focal plane. Now we construct the optical center, for that will solve the problem. So construct it, let two chords of the ellipse through the point P be XY, ZW. $XZ \cap WY$ and $YZ \cap XW$ are the images of two points at infinity in perpendicular directions, so the optical center lies on the circle with them as a diameter. Similarly making any other quadrilateral gives the optical center's position.