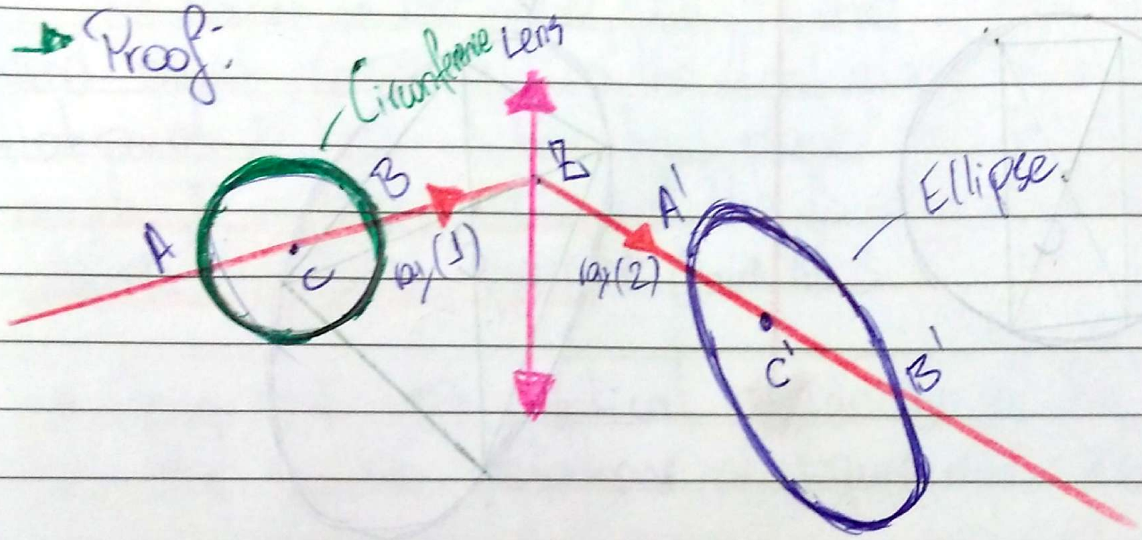
Physics Cup - Problem 4 - Diogo Correia Netto

• Lets start our solution with the following Claim:

Claim No.1

→ Points A' and B' that lie oppositely to the center of the ellipse are diametrically opposite in the original circunference.

→ Proof:

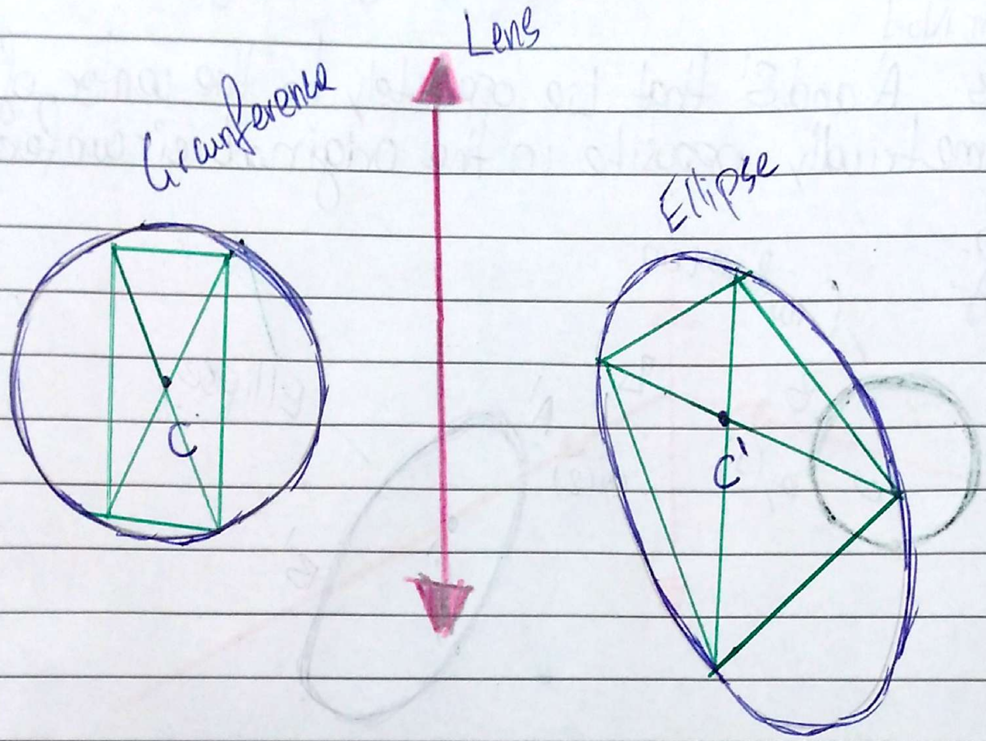


Consider the ray (1) in the figure, that intersects the lens in point Z. As C' is the image of C, the refracted ray (2) has to pass through C'.

Now, it should be noted that if ray (1) passes through points A and B, the intersections of ray (2) and the ellipse must give the images A' and B' of these points, as these images necessarily lie in the ellipse.

By reversibility we conclude that pairs of opposite points such as A' and B' must be the images of diametrically points in the circunference (such as A and B)

→ Now lets consider a non trapezium quadrilateral with diagonals that pass through point C' (image of the center of the circumference)



According to Claim 1 opposite points in the ellipse must be diametrically opposite in the circunference. Note that this fact implies that both the diagonals of the original quadrilateral are diameters ⇒ The original quadrilateral (the one inscribed in the circunference) is a rectangle. → Claim No. 2

→ Given Claim No. 1 and Claim No. 2 we have transformed our original problem into the following one:

- New problem: Given a quadrilateral known to be the image of a rectangle as generated by a lens, reconstruct the position of the lens.

→ Lets solve the new problem:

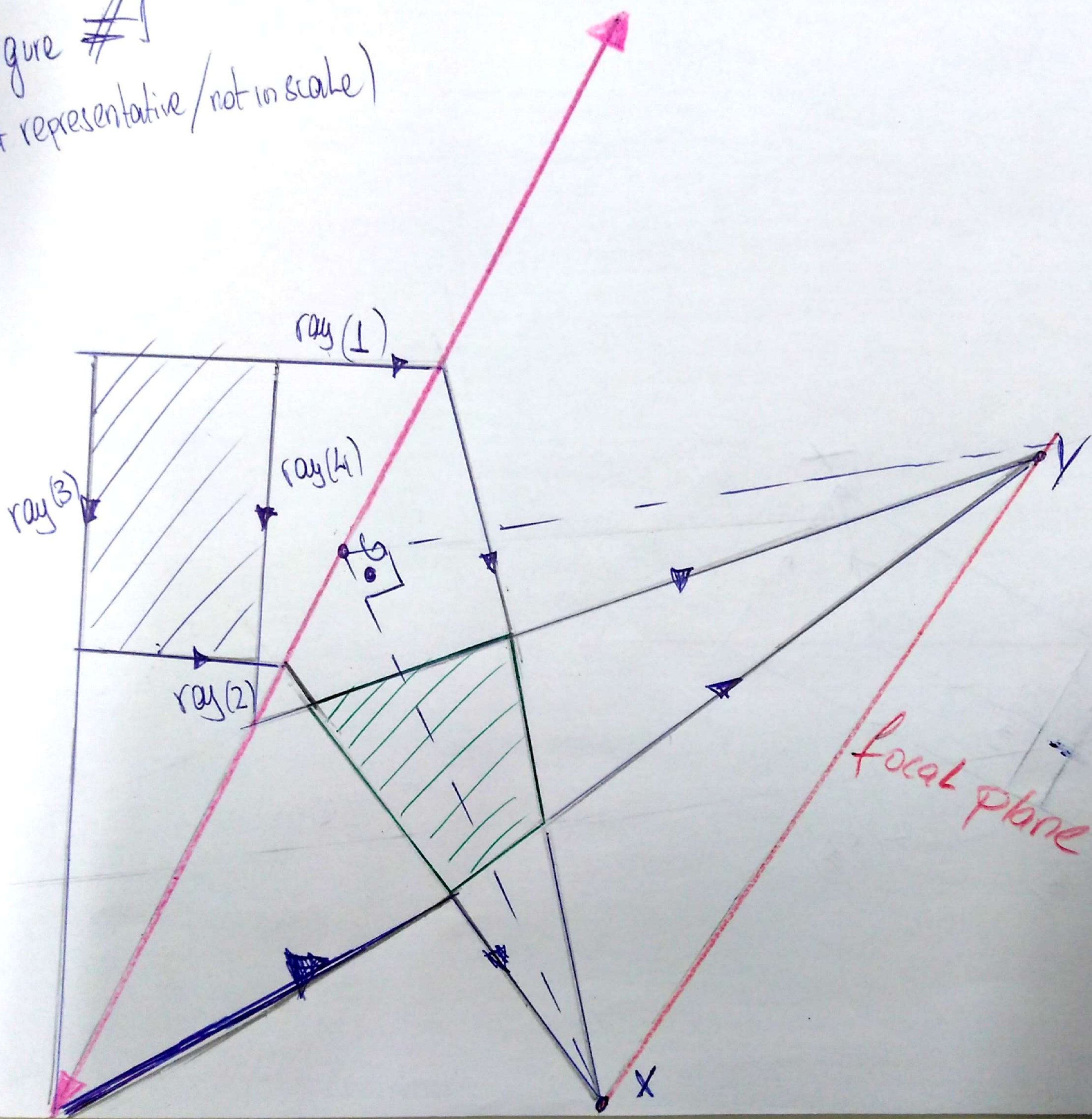
* Claim No.3: The intersections of opposite sides of the quadrilate define the focal plane of the Lens.

Proof:

As shown in the figure, rays (1) and (2) ~~are~~ are parallel and must converge in the focal plane. From this, we conclude that point X Lies in the focal plane. For the same reason, point Y must also lie in the focal plane ⇒ $\boxed{\text{The Line XY defines the focal plane}}$ → Claim No.3

Figure #1

(just representative / not in scale)

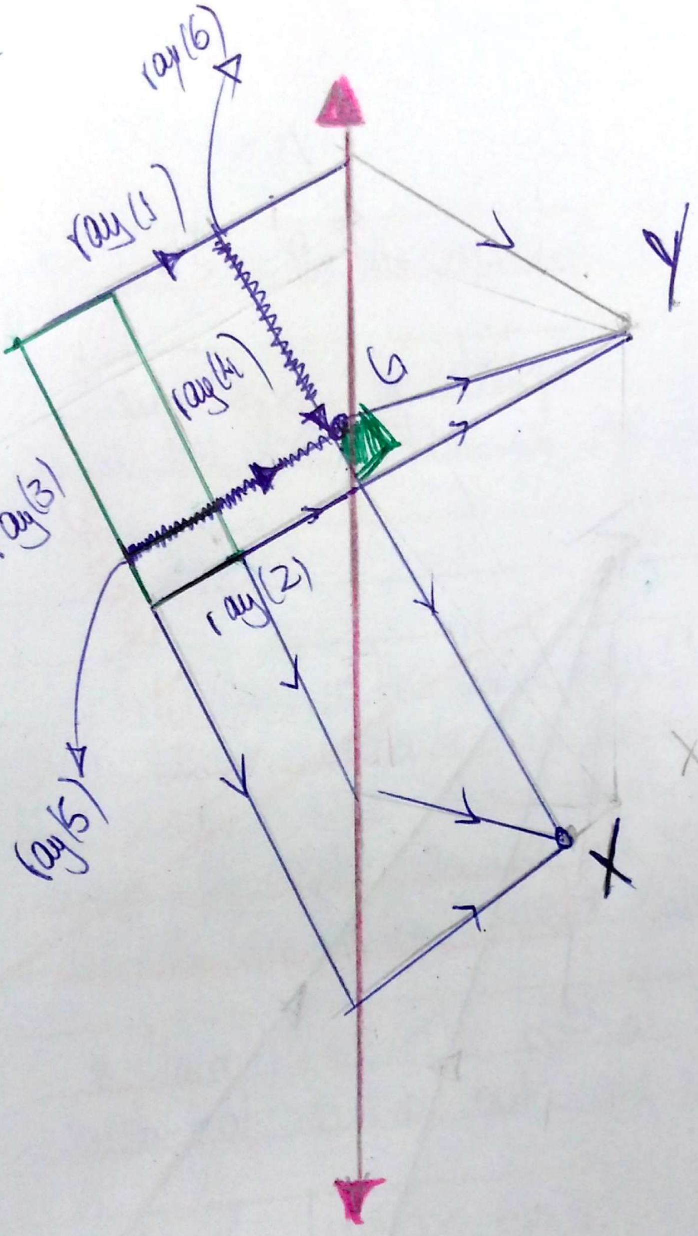


* Claim No.4: The optical center G of the Lens is such that in the notations of figure #1 $X\hat{G}Y = 90°$

Proof: Consider rays (5) and (6) such that 6 is parallel to 3 and 4 and 5 is parallel to (1) and (2) And (5) and (6) pass through the optical center of the Lens.
As the sides of the rectangle are perpendicular, it follows that $X\hat{G}Y = 90°$.

As a consequence, G must Lie in the circumference w_1 that has XY as a diameter.

Figure 2
(Not in scale)



→ Finally, to conclude our problem we describe the algorithm to find the lens:

1. Obtain a non trapezium quadrilateral Q_1 that is inscribed in the ellipse and has diagonals that pass through C'.

2. Obtain the points ~~X_1 and Y_2 by the intersection of~~ X_1 and Y_1 by the intersection of the opposite sides of Q_1. X_1Y_1 defines the focal plane.

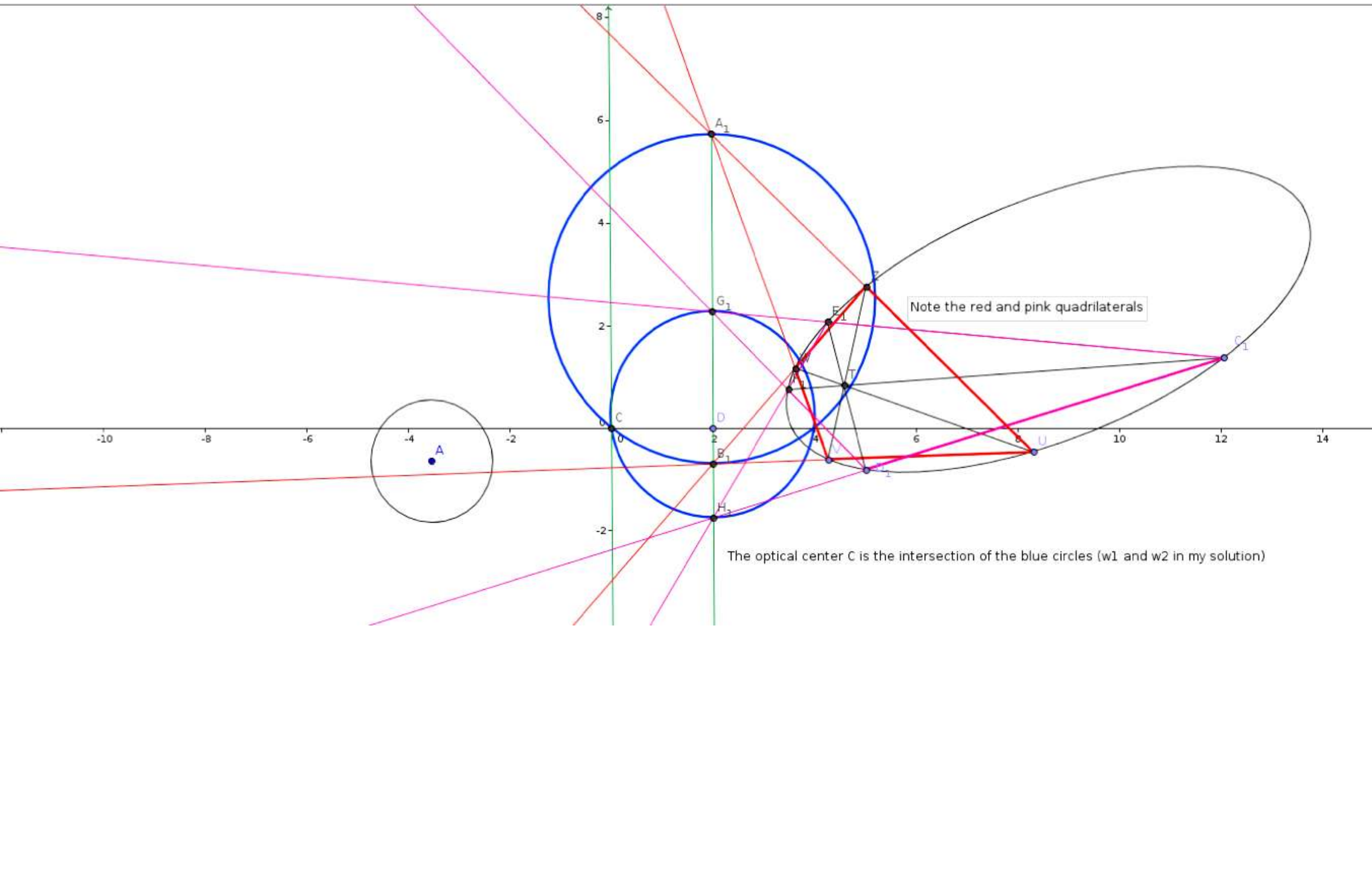
3. Draw a circumference w_1 that has X_1Y_1 as a diameter.

4. Repeat the process for a different non trapezium quadrilateral Q_2.

5. Draw the corresponding circumference w_2 of Q_2

6. The optical center of the Lens is the intersection of w_1 and w_2. The direction of the Lens is parallel to the focal plane already described.

Here is a Geogebra figure that ilustrates the construction


Note the red and pink quadrilaterals
The optical center C is the intersection of the blue circles (w1 and w2 in my solution)