I'll send my solution to the Physics Cup problem 4 using only compass-and-straightedge construction.

Solution:

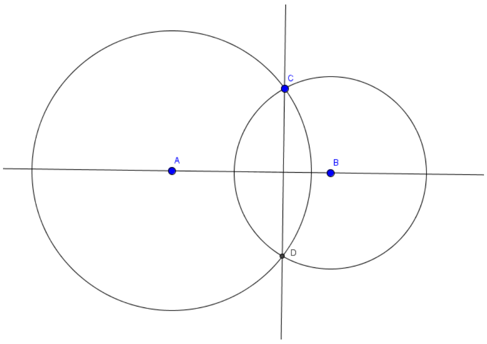



Figure no. 1 - How to draw a line through point C (not on line AB) that is perpendicular to the given line AB. Since CD is the radical axis of the two circles, it is perpendicular to the line connecting the centers (AB).

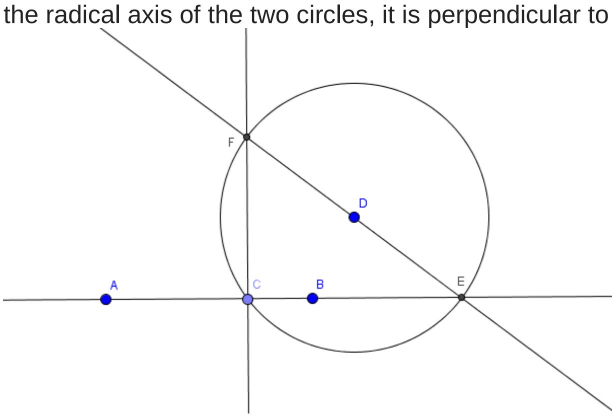



Figure no. 2 - How to draw a line throught point C (on line AB) that is perpendicular to the given line AB. (Using Thales' theorem).

We can draw a parallel line by drawing two perpendicular lines.

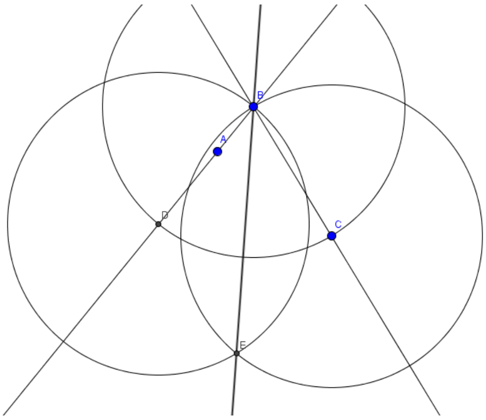
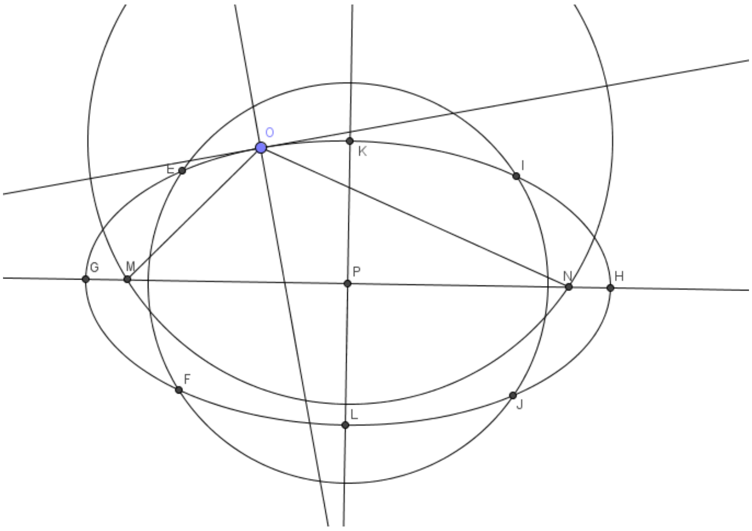



Figure no. 3 - How to draw an angle bisector. (BE bisects angle ABC)



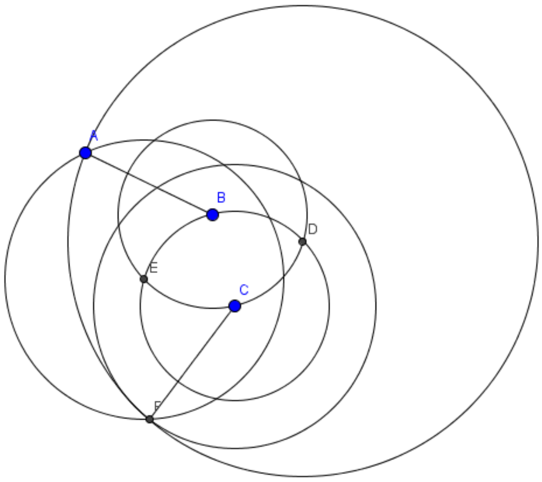




Figure no. 4 - How to use a collapsing compass as a non-collapsing compass. (If given a segment AB, how to draw a circle centered at point C with radius R=|AB|). Note that F is the reflection of point A about the perpendicular bisector of BC.

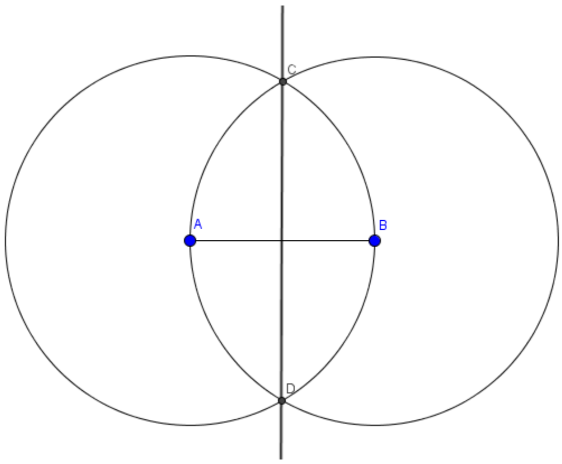



Figure no. 5 - How to find the midpoint and the perpendicular bisector of a given segment AB.

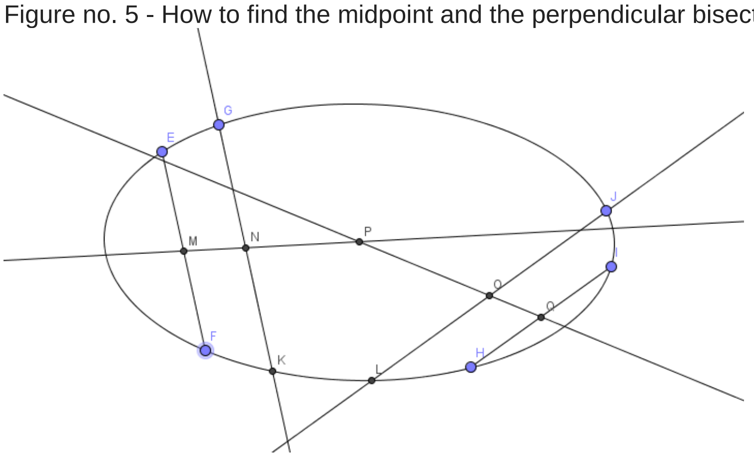



Figure no. 6 - How to find the center of a given ellipse. We can do that by drawing two pairs of parallel chords and their midpoints. The lines drawn through respective midpoints for both pairs meet at the center of the ellipse (since a line drawn through the midpoints of two parallel chords is a diameter and passes through the center). In the figure center is denoted by P.

Figure no. 7 - How to find the axes and the foci of an ellipse and how to find a tangent line from a point on an ellipse. At first we draw a circle centered at the center of an ellipse that intersects the ellipse at four points. The four points form a rectangle and due to symmetry the axes are the perpendicular bisectors of the sides of the rectangle (axes are GH and KL). Then from the definition of an ellipse by drawing a circle centered at the point K (endpoint of the minor axis) with radius equal to GP (semi-major axis), the intersection points of the circle and the major axis are the foci of the ellipse (M and N). To find the tangent line through point O, we draw two lines MO and NO and by the property of an ellipse the bisector of angle MON is the normal line at point O. The tangent line is therefore the perpendicular line of the angle bisector.

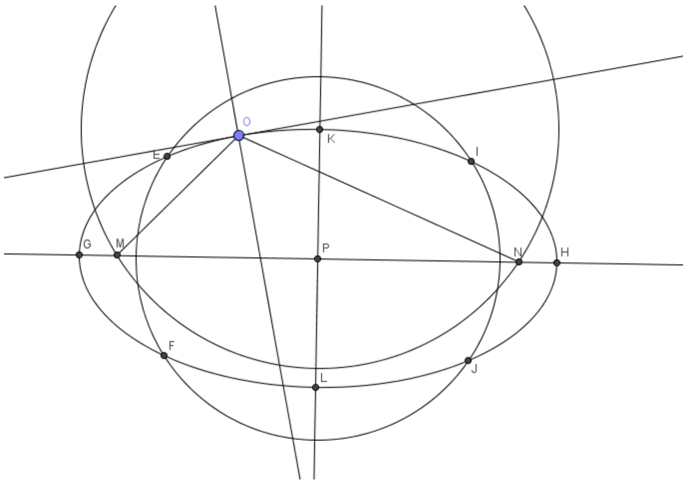
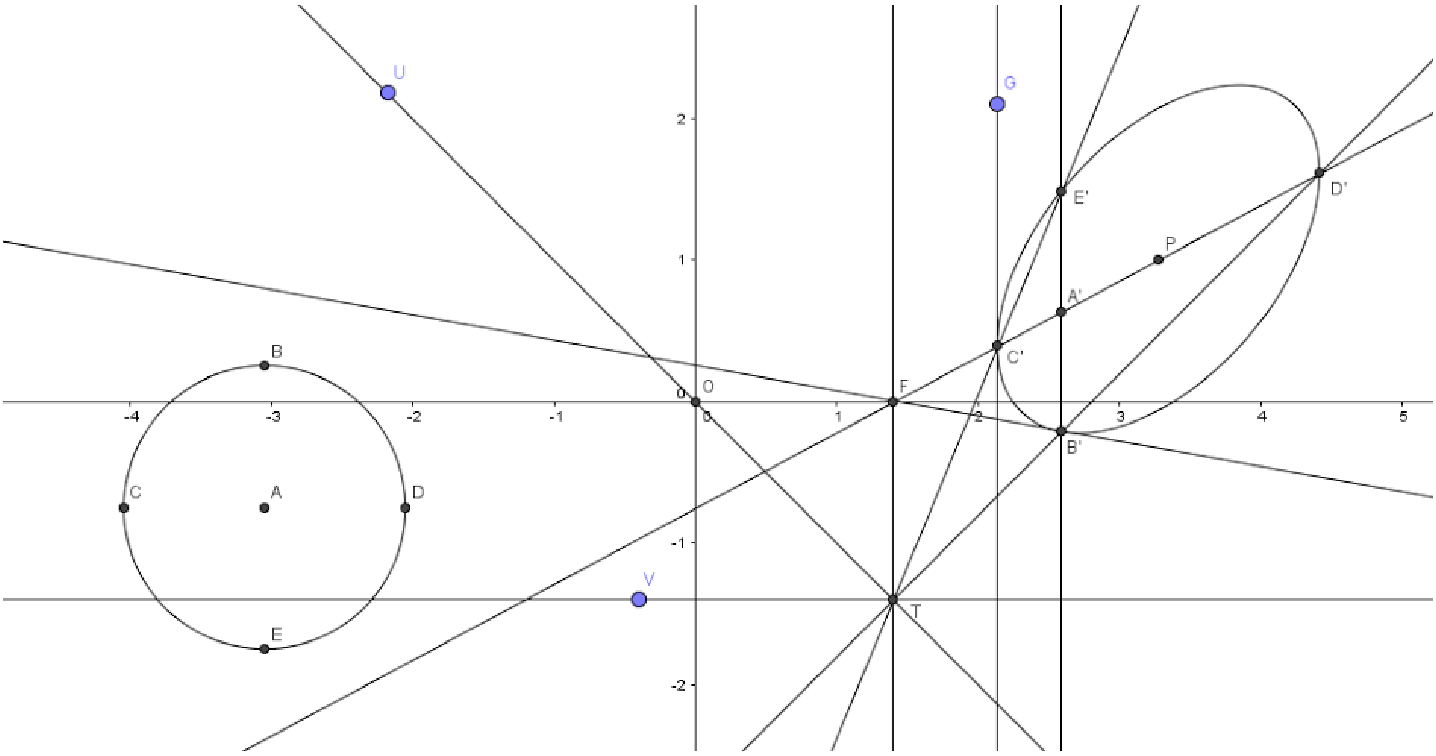



Figure no. 8 - The main solution.




Let the lens be centered at O(0;0) and be parallel with y-axis. Assume WLOG that the ellipse is in the positive x area, then the original circle is in the negative x area. Let A be the center of the circle and let B, C, D, E be points on the circle with maximum y value, minimum x value, minimum y value, maximum x value, respectively. Let A' - E' be images of these points. Obviously BCDE is a square. Since the image of a segment (that doesn't intersect with the focal plane) is a segment, then C'D' passes through A' (since CD passes through A). Also E'B' passes through A'. Since 1/-(x coordinate of object) + 1/(x coordinate of image) = 1/f, then minimum x coordinate of the object corresponds to minimum x coordinate of the image and vice versa. Therefore C' is the point on the ellipse which has the lowest x value and D' has respectively the highest x value. Due to symmetry, the line connecting these two extreme points passes through the center of the ellipse (P). Therefore C', A', P and D' lie on the same line and we can construct C' and D' by drawing the line A'P and marking the intersection points with the ellipse. Let C'G be tangent to the ellipse. Since C' is the point with minimum x value, the tangent (C'G) is parallel to the y-axis. Since A, B and E have the same x value, then by the lens formula A', B', E' also have the same x value. We can construct E' and B' by drawing a line through A' that is parallel to C'G. It is obvious that the image of a tangent of the circle is also tangent line of the ellipse. Since the tangent line from the point B and the line CD are parallel, then the rays after passing through the lens intersect at the focus of the lens (F). Therefore tangent line drawn from point B' and C'D' intersect at the focus (F). We can draw the optical axis (x-axis), which is perpendicular to C'G and passes through F. Let UO be a line parallel to BD. Since parallel rays after refraction meet at the same point in the focal plane, then UO, B'D' and C'E' meet at one point (the ray UO doesn't refract because it passes through the center of the lens and BD and CE are parallel to UO). Let the point be T (intersection of B'D' and C'E'). Since BD forms a 45 degree angle with x and y-axis (side of a square), then we can costruct line UO by drawing a perpendicular to TF (let it be TV) and then drawing the bisector of the angle VTF, which is also parallel to BD and therefore is the same line as UO. The line intersects the optical axis at point O. The perpendicular line of the optical axis through point O is the position of lens.

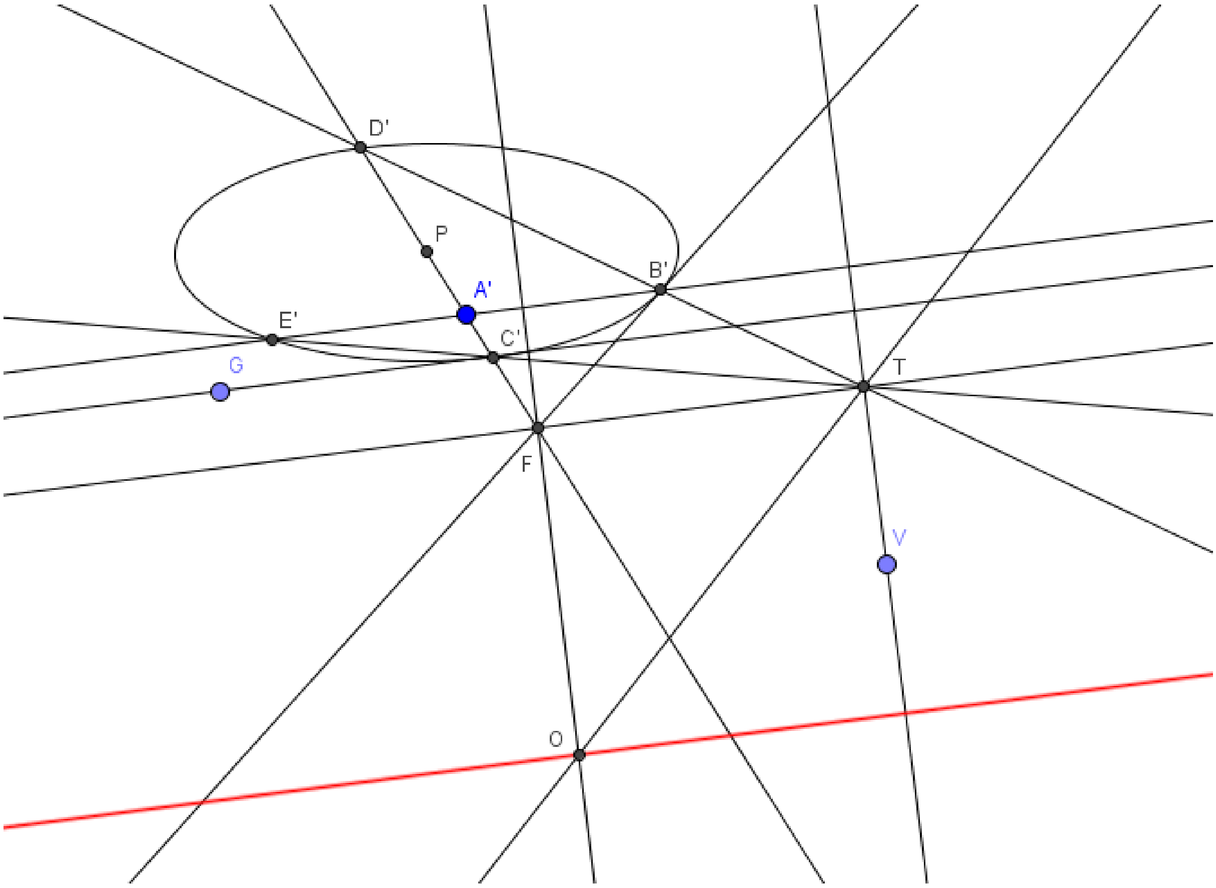


Figure no. 9 - The same construction for an ellipse similar to the given ellipse. The lens is denoted by the red line.