

Physics Cup 2017 - Problem 2 (contributed by Ivan Erofeev) without hints. 28th April 2017

There are n points forming n -gon. Adjacent points (sides of n -gon) are connected with resistance $R_s = 2\Omega$, all other pairs (diagonals) are connected with resistance $R_d = 1\Omega$. Find the full resistance between two adjacent points. Using your expression, calculate answer (as a rational number in ohms) for $n = 3$, $n = 4$, and $n = 5$. Feel free to use mathematics software to ease algebraic calculations, if needed (such as Wolfram Alpha).

Hints: *First*, the problem can be solved using a brute-force method by writing down all the Kirchoff's laws and simplifying the set of equations, or by finding a simpler equivalent circuit.

Second, in both cases you'll end up in a recurrence equation for a series of unknown quantities x_k in the form $x_{k+1} = ax_k + bx_{k-1}$, where a and b are constants. This equation is solved in the same way as linear differential equations with constant multipliers: we seek for the solution in the form $x_k = \lambda^k$. This recurrence equation is linear, so any linear combination of solutions is also a solution, and the coefficients entering a linear combination is to be found from the additional conditions (e.g. known values for x_1 and x_n).

Third, if you want to avoid the brute force approach to a certain extent, study problems 48 and 49 from <http://www.ioc.ee/~kalda/ipho/electricity-circuits.pdf>, and the hints provided for these problems.

Results thus far (by the order of submission):

Marco Malandrone: 2.5937

Siddharth Tiwary: 2.3579

Akihiro Watanabe: 2.1436

Dylan Toh: 1.9487

Elene Kravishvili: 1.7715

Jacob Teo: 1.6105

Diogo Netto: 1.4641

Non-official participants (by the order of submission):

Taavet Kalda: 2.3579