Physics Cup 2017 - Problem 2 (contributed by Ivan Erofeev) without hints. 25th May 2017

There are n points forming n-gon. Adjacent points (sides of n-gon) are connected with resistance $R_s = 2\,\Omega$, all other pairs (diagonals) are connected with resistance $R_s = 1\,\Omega$ Find the full resistance between two adjacent points. Using your expression, calculate answer (as a rational number in ohms) for $n = 3$, $n = 4$, and $n = 5$. Feel free to use mathematics software to ease algebraic calculations, if needed (such as Wolfram Alpha).

Hints: *First,* the problem can be solved using a brute-force method by writing down all the Kirchoff's laws and simplifying the set of equations, or by finding a simpler equivalent circuit.

Second, in both cases you'll end up in a recurrence equation for a series of unknown quantities x_k in the form $x_{k+1} = ax_k + bx_{k-1}$, where a and b are constants. This equation is solved in the same way as linear differential equations with constant multipliers: we seek for the solution in the form $x_k = \lambda^k$. This recurrence equation is linear, so any linear combination of solutions is also a solution, and the coefficients entering a linear combination is to be found from the additional conditions (e.g. known values for x_1 and x_n).

Third, if you want to avoid the brute force approach to a certain extent, study problems 48 and 49 from http://www.ioc.ee/~kalda/ipho/electricity-circuits.pdf, and the hints provided for these problems.

Knowing the resistance of a polygon where all the diagonals and all the resistances are equal (see the previous hint), you should be able to reduce this problem to the problem of finding the resistance of a wheel graph, cf. https://en.wikipedia.org/wiki/Wheel_graph, where the "spokes" and "rim segments" have different resistances. Now you should be able to write down Kirhoff's laws in such a way that you can use the hint No 2.

Results thus far (by the order of submission):
Marco Malandrone: 2.5937
Siddharth Tiwary: 2.3579
Akihiro Watanabe: 2.1436
Dylan Toh: 1.9487
Elene Kravishvili: 1.7715
Jacob Teo: 1.6105
Alkin Kaz: 1.4641
Diogo Netto: 1.331
Victor Almeida Ivo: 1.694
Gabriel Golfetti: 1.1

Non-official participants (by the order of submission):
Taavet Kalda: 2.3579