Physics Cup 2017 - Problem 3. 23rd April 2017

Consider a sparse electron gas at temperature $T = T_0$ in homogeneous magnetic field. Initially, the magnetic flux density is B_0, and is increased up to $B = 2B_0$; during the process, the growth rate of the B-field is kept very small, $\frac{\mathrm{d}B}{\mathrm{d}t} \ll \frac{B^2 e}{m}$, where e denotes the elementary charge, and m — the electron mass. Let us define the characteristic time τ of this process as the typical value of $\left(\frac{\mathrm{d}\ln B}{\mathrm{d}t}\right)^{-1}$; then, the above given condition can be rewritten as $\tau \gg \frac{m}{Be}$. The sparseness of the electron gas is described by the mean free path length λ of electrons, and by the mean time interval between collisions $t \approx \lambda\sqrt{\frac{m}{kT}}$. Here we assume that the gas is so sparse that $t \gg \tau$. Upon reaching the value $B = 2B_0$, the B-field strength is kept constant for a very long time $\mathcal{T} \gg t$, and is later slowly decreased back to the original value $B = B_0$ (the characteristic time of the process is again τ). Now, if we wait until thermal equilibrium is reached, what will be the temperature T' of the electron gas?

Keep in mind that if a quantum-mechanical system is slowly (*adiabatically*) perturbed, it retains its quantum state if the characteristic time of the process is much longer than $\hbar/\Delta E$, where ΔE is the energy level difference between neighbouring states.