Physics Cup 2017 - Problem 3. 14th May 2017

Consider a sparse electron gas at temperature $T = T_0$ in homogeneous magnetic field. Initially, the magnetic flux density is B_0, and is increased up to $B = 2B_0$; during the process, the growth rate of the B-field is kept very small, $\frac{\mathrm{d}B}{\mathrm{d}t} \ll \frac{B^2 e}{m}$, where e denotes the elementary charge, and m — the electron mass. Let us define the characteristic time τ of this process as the typical value of $\left(\frac{\mathrm{d}\ln B}{\mathrm{d}t}\right)^{-1}$; then, the above given condition can be rewritten as $\tau \gg \frac{m}{Be}$. The sparseness of the electron gas is described by the mean free path length λ of electrons, and by the mean time interval between collisions $t \approx \lambda\sqrt{\frac{m}{kT}}$. Here we assume that the gas is so sparse that $t \gg \tau$. Upon reaching the value $B = 2B_0$, the B-field strength is kept constant for a very long time $\mathcal{T} \gg t$, and is later slowly decreased back to the original value $B = B_0$ (the characteristic time of the process is again τ). Now, if we wait until thermal equilibrium is reached, what will be the temperature T' of the electron gas?

Keep in mind that if a quantum-mechanical system is slowly (*adiabatically*) perturbed, it retains its quantum state if the characteristic time of the process is much longer than $\hbar/\Delta E$, where ΔE is the energy level difference between neighbouring states.

Hints: *First,* Notice that when the magnetic field is being changed, the kinetic energy of electrons associated with the motion along the field remains unchanged. Meanwhile, the kinetic energy of electrons associated with the motion perpendicularly to the field changeds, and the change can be found from the condition that the order number of its quantum-mechanical energy level remains constant.

Second, quantum mechanical energy levels can be found by applying quasi-classical approach to the cyclotron orbits of the electrons.

Extended hints: Notice that when the magnetic field is increased, the energy levels for the motion of electron in the perpendicular to the field direction grows, but the level occupancy probabilities remain unchanged, hence you can deduce from the Boltzmann distribution the temperature change.

To sum up, there are the following steps in the evolution of the electron gas: first, adiabatic expansion, the occupancy of the quantum-mechanical levels of electrons in circular cyclotron orbits remains constant; hence, as it follows from the Boltzmann distribution, the temperature associated with the perpendicular-to-the-field motion is proportional to the energy level difference (magnetic field has no effect on the longitudinal motion and hence, the longitudinal temperature remains constant). Second, slow heat exchange (due to collisions) between longitudinal and perpendicular motions, thermal equilibrium described by a single temperature is reached. Third, the whole process described up til now is repeated, with the only difference that now magnetic field is decreased.

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