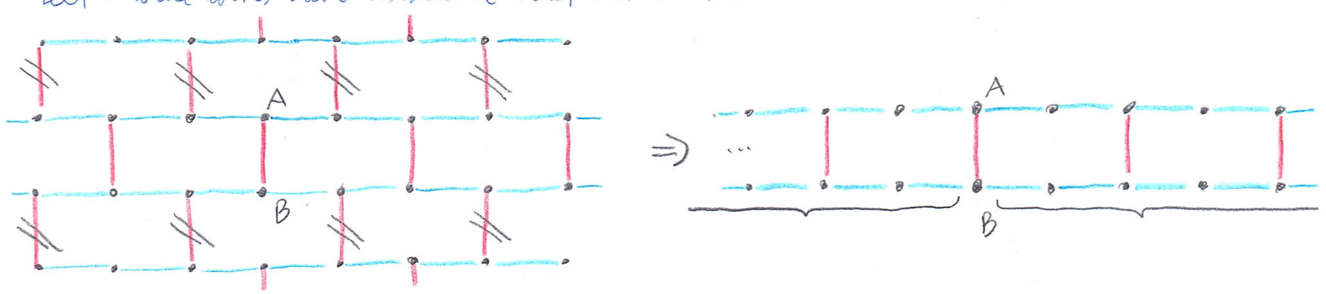
- To obtain upper and lower limits for the resistance I will use the following theorems:
  (I) For an arbitrary circuit which consists of resistors and has two leads, A and B, if a current I is driven into the lead A and out from lead B, the current distributes between the the resistors of the circuit so as to minimize the overall power dissipation. In other words, the power dissipation of the actual current distribution is always smaller compared to any fictious current distribution satisfying only the Kirchoff's law of currents
  (II) For the same circuit, if there is a voltage drop V between leads A and B, the voltage distributes ~~over~~ between the nodes of the circuit so as to minimize the overall power dissipation. In other words, the power dissipation of the actual voltage distribution is always smaller compared to any fictious voltage distribution violating the Kirchoff's law of currents.

As a conclusion of these theorems, cutting off a wire will increase the resistance, and short-circuiting ~~with~~ a wire will decrease the resistance. [1]

- <u>Upper limit for the resistance:</u>
  - Let's cut off wires as indicated by the figure below (I use the same colour code as the problem's text: blue wires have resistance $1\,\Omega$, red wires have resistance $2018\,\Omega$).



  - Let us determine the resistance R of the infinite periodic chain of resistors that appeared in the new circuit (see also below). This can be done by making use of the self-similarity of the chain: removal of the first period does not change its resistance.

Hence we can write equality

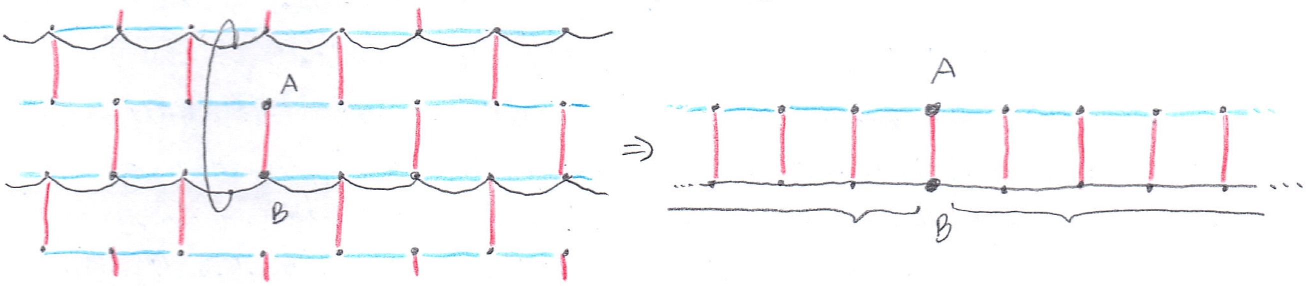
$$(1)\quad 4+\frac{2018R}{2018+R}=R \Leftrightarrow R^2-4R-8072=0$$

Equation (1) has only one positive root, which is $R=2(1+\sqrt{2019})$ (ohm)

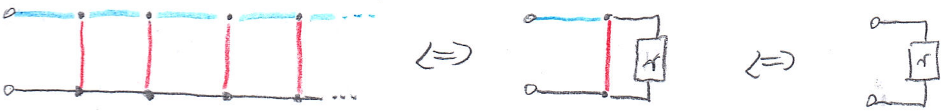
- The resistance ϱ_u between nodes A and B can be calculated as the ~~the~~ net resistance of three ~~parallelly connected~~ resistors ($R, R, 2018\,\Omega$) connected in parallel:

$$(2)\quad \varrho_u=\frac{1}{\frac{1}{R}+\frac{1}{R}+\frac{1}{2018}}=\frac{2018}{\sqrt{2019}}\ (\Omega)$$

- <u>Lower limit for the resistance:</u>
  - Let us short-circuit wires as indicated by the figure below. (black wires have zero ~~no~~ resistance)



  - What follows is similar to what we did ~~as~~ beforehand. Firstly, we determine the resistance r ~~(R)~~ of the infinite chain of resistors (see below).



We can write

$$(3)\quad 1+\frac{2018\,r}{2018+r}=r \iff r^2-r-2018=0$$

The only positive root of (3) is $r=\frac{1}{2}\left(1+\sqrt{8073}\right)\ (\Omega)$

  - The resistance S_ℓ between nodes A and B now can be calculated as the net resistance of three resistors (r, r, $2018\,\Omega$) connected in parallel:

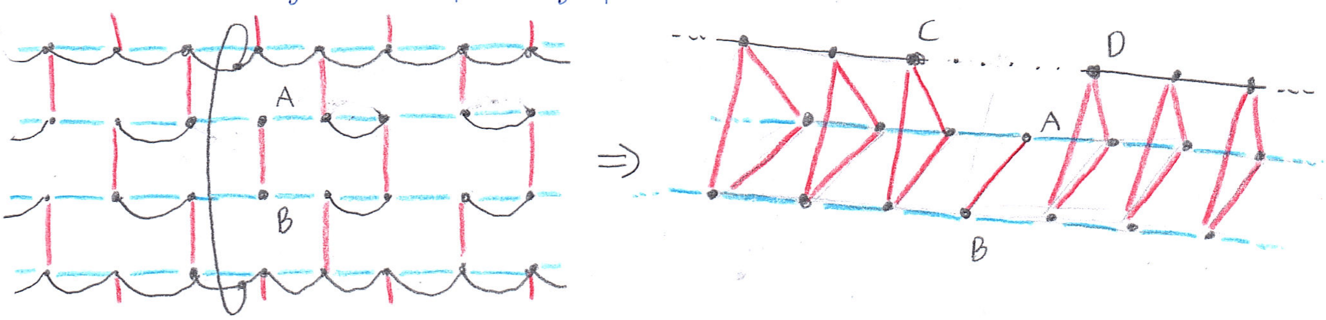
$$(4)\quad S_\ell=\frac{1}{\frac{1}{r}+\frac{1}{r}+\frac{1}{2018}}=\frac{2018}{\sqrt{8073}}\ (\Omega)$$

- <u>Conclusion:</u>
  - We proved, that for the resistance S between nodes A and B in the original circuit the following inequality holds: $S_\ell < S < S_u$,

    where $S_\ell=\frac{2018}{\sqrt{8073}}\,\Omega\approx 22{,}46\,\Omega$ and $S_u=\frac{2018}{\sqrt{2019}}\,\Omega\approx 44{,}91\,\Omega$

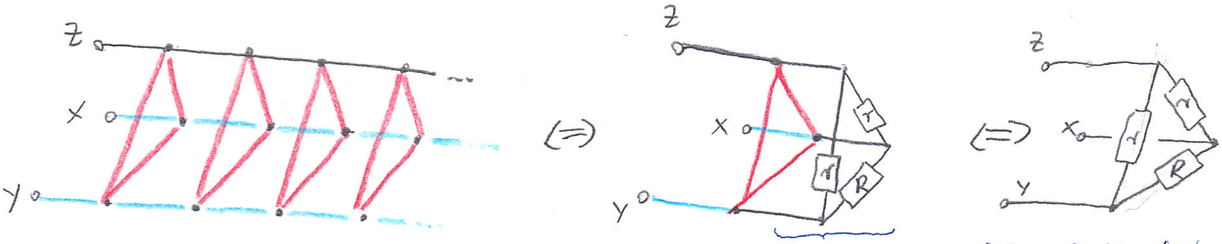
  - The restriction $\frac{S_u}{S_\ell}\leq 2$ is satisfied, as $\frac{S_u}{S_\ell}=\sqrt{\frac{8073}{2019}}=\sqrt{4-\frac{1}{673}}<\sqrt{4}=2$

- <u>Better lower limit for the resistance:</u>
  - If we short-circuit „less" wires ~~as we did~~ than in the first case, we gain a better lower limit — although the complexity of mathematical calculations increases.



Points C and D are equipotential because of the symmetry of the circuit (the current is driven into lead A and out from lead B), so the wire between them can be cut off.

- Now we have an infinite chain of resistors again, but this time with three ports (see below). However, the net resistance R_{xy} can be found the same way as ~~we did~~ before owning to the fact that any circuit which consists only of resistors and has three ports is equivalent to Δ- or a Y- connection of three appropriately chosen resistors. [2]
Let the equivalent Δ connection consists of resistors with resistance r, r, R (here we made use of the chain's symmetry).



equivalent circuit of the infinite chain

We can write equalities

$$(5)\quad R_{xy} = 2 + \frac{\frac{2018R}{2018+R}\cdot 2\cdot\frac{2018r}{2018+r}}{\frac{2018R}{2018+R} + 2\cdot\frac{2018r}{2018+r}} = \frac{R\cdot 2r}{R+2r}$$

$$(6)\quad R_{xz} = R_{yz} = 1 + \frac{\frac{2018r}{2018+r}\cdot\left(\frac{2018R}{2018+R} + \frac{2018r}{2018+r}\right)}{\frac{2018r}{2018+r} + \left(\frac{2018R}{2018+R} + \frac{2018r}{2018+r}\right)} = \frac{r\cdot(R+r)}{r+(R+r)}$$

- I used MATHCAD (14.0) to solve the system of equations above, the results are:

$$R = \frac{10611586843}{106853100} \approx 99{,}31\ (\Omega) \quad \text{and} \quad r = \frac{101959}{1059} \approx 96{,}28\ (\Omega)$$

So $R_{xy} = \dfrac{R\cdot 2r}{R+2r} = \dfrac{208154}{3177} \approx 65{,}52\ (\Omega)$

- The resistance between nodes A and B now can be calculated as the net resistance of three resistors (having resistance R_{xy}, R_{xy}, $2018\,\Omega$) connected in parallel:

$$(7)\quad \rho_e' = \frac{1}{\frac{1}{R_{xy}} + \frac{1}{R_{xy}} + \frac{1}{2018}} = \frac{210027386}{6515263} \approx 32{,}236\ (\Omega) \quad \text{(here we rounded down)}$$

- Thus ρ_e' is a better lower limit for the resistance of the original ~~(infinite)~~ circuit than the one we found previously.

- References:

[1] Jaan Kalda, Electrical circuits (pages 11-12, 15-16)
[2] Jaan Kalda, Electrical circuits (pages 5, 14-15)
(Retrieved from https://www.ioc.ee/~kalda/ipho/electricity-circuits.pdf)