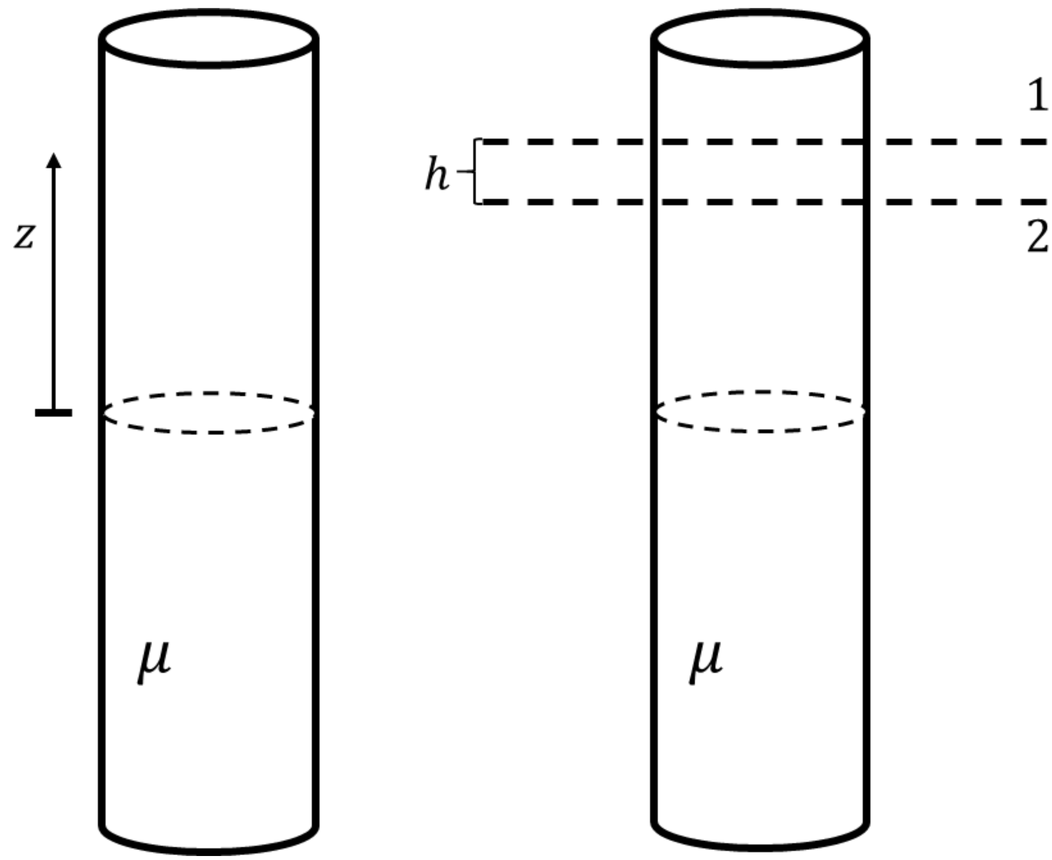
Using Boundary conditions at the interface($r = R$), we have(Note proof in Appendix 2):

$$B_r^{in} = B_r^{out} \text{ and also } \frac{B_z^{in}}{\mu} = B_z^{out} \quad (1)$$

Since $\mu \gg 1$, $B_z^{in} \gg B_z^{out}$, therefore the magnetic flux of B_z inside the cylinder is much larger than the flux of B_z outside the cylinder.

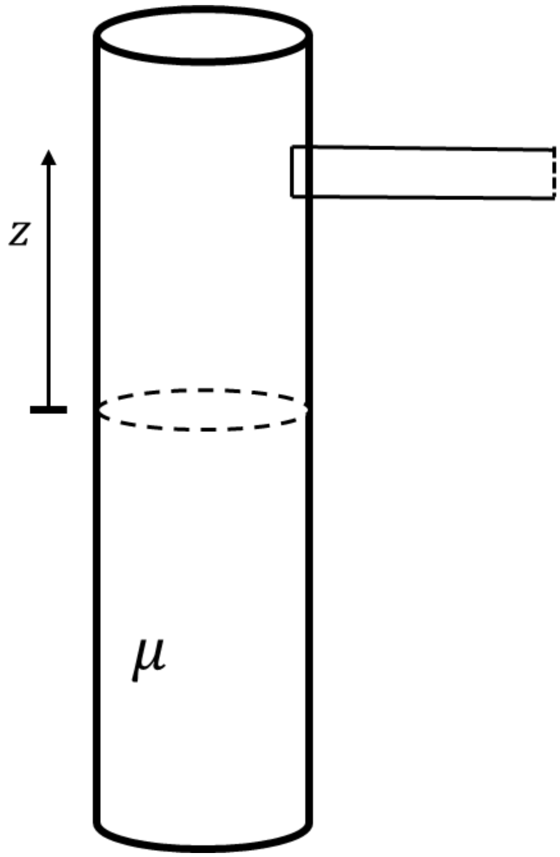
This manner using Gauss law between the infinite planes 1-2 in the figure below:



We obtain therefore(using that B_z^{in} does not depend on r, note proof in appendix 1):

$$B_z^{in}(z)\pi R^2 = 2\pi r h B_r^{out} + B_z^{in}(z+h)\pi R^2 \Leftrightarrow 2rB_r^{out} = -R^2\frac{dB_z^{in}}{dz} \quad (1)$$

Now, using Amperes law for the loop below we have:



$$\int \vec{H} \cdot d\vec{l} = I_{enclosed} = 0 \Leftrightarrow H_z^{in} h + \int_R^{\infty} \left(H_r^{out}(z+h) - H_r^{out}(z)\right) dr = 0 \quad (2)$$

Simplifying (2), we obtain therefore(using that $\vec{B} = \mu\mu_0\vec{H}$):

$$H_z^{in} = -\int_R^{\infty} \frac{dH_r^{out}}{dz} dr \Leftrightarrow \frac{B_z^{in}}{\mu} = -\int_R^{\infty} \frac{dB_r^{out}}{dz} dr \quad (3)$$

Now using (1) we obtain:

$$\frac{dB_r^{out}}{dz} = -\frac{R^2}{2r} \frac{d^2 B_z^{in}}{dz^2} \quad (4)$$

Using (4) in (3), we have now(defining the divergent logarithmic integral as $\eta = \int_R^{\infty} \frac{dr}{r}$) :

$$\frac{B_z^{in}}{\mu} = \frac{R^2}{2} \frac{d^2 B_z^{in}}{dz^2} \int_R^{\infty} \frac{dr}{r} \Leftrightarrow \frac{d^2 B_z^{in}}{dz^2} = \frac{2}{\mu\eta R^2} B_z^{in} \quad (5)$$

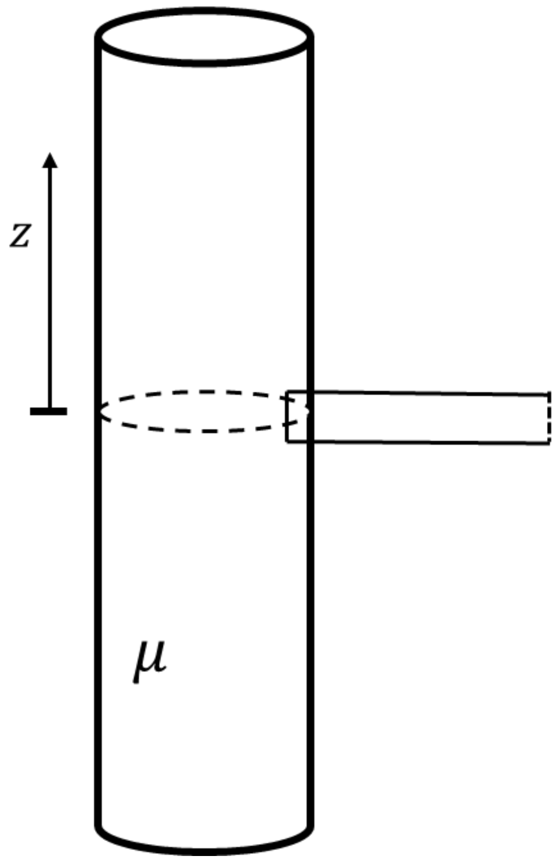
Therefore the solutions for this equation are(Valid only for $z > 0$):

$$B_z^{in} = Ae^{-z/\lambda}, where\ \lambda = R\sqrt{\frac{\mu\eta}{2}} \quad (6)$$

In (6), one can see that $\lambda \gg R$, since $\mu, \eta \gg 1$. Now, using (6) and (1), we obtain that:

$$B_r^{out} = \frac{AR^2}{2\lambda r} e^{-z/\lambda} \ (7)$$

Now, to obtain A, we again use Amperes law, for the loop below:



This time, we have(using that the loop is at $z = 0$):

$$\int \vec{H} \cdot d\vec{l} = I_{enclosed} = I \Leftrightarrow 2 \int_R^\infty B_r^{out} dr = \mu_0 I \Leftrightarrow A = \frac{\mu_0 I \lambda}{R^2 \eta} \ (8)$$

Remember that η is defined as $\eta = \int_R^\infty \frac{dr}{r}$.

This manner we obtain our expression for B_z^{in}:

$$B_z^{in} = \frac{\mu_0 I \lambda}{R^2 \eta} e^{-z/\lambda} \ (9)$$

Therefore the magnetic flux through the loop is:

$$\phi = \pi R^2 B_z^{in}(z = 0) = \frac{\mu_0 I \lambda \pi}{\eta} \ (10)$$

Since inductance is defined as $L = \phi/I$, we obtain therefore:

$$L = \frac{\mu_0 \lambda \pi}{\eta} \quad (11)$$

Now let us use the fact that $\lambda \gg R$ to find a expression for L. Since $\eta = \int_R^\infty \frac{dr}{r}$, and $\lambda \gg R$, therefore $\eta = \int_R^{\beta\lambda} \frac{dr}{r} = \ln\left(\frac{\beta\lambda}{R}\right)$, where β is a constant that we can adjust, since $\lambda = R\sqrt{\frac{\mu\eta}{2}}$ we obtain therefore that:

$$\eta = \ln\left(\beta\sqrt{\frac{\mu\eta}{2}}\right) = \ln\beta + \frac{1}{2}\ln\left(\frac{\mu\eta}{2}\right) \Leftrightarrow 2\eta = \ln\beta^2 + \ln\left(\frac{\mu\eta}{2}\right) = ln\mu + ln\eta + ln\beta^2 - ln2 \ (12)$$

Now, since $ln\mu \gg ln2$ and also that $2\eta \gg ln\eta$, and that by hypothesis μ is arbitrarily large(so we can ignore $ln\beta^2$) we have that (12) simplifies to:

$$2\eta = ln\mu \ (13)$$

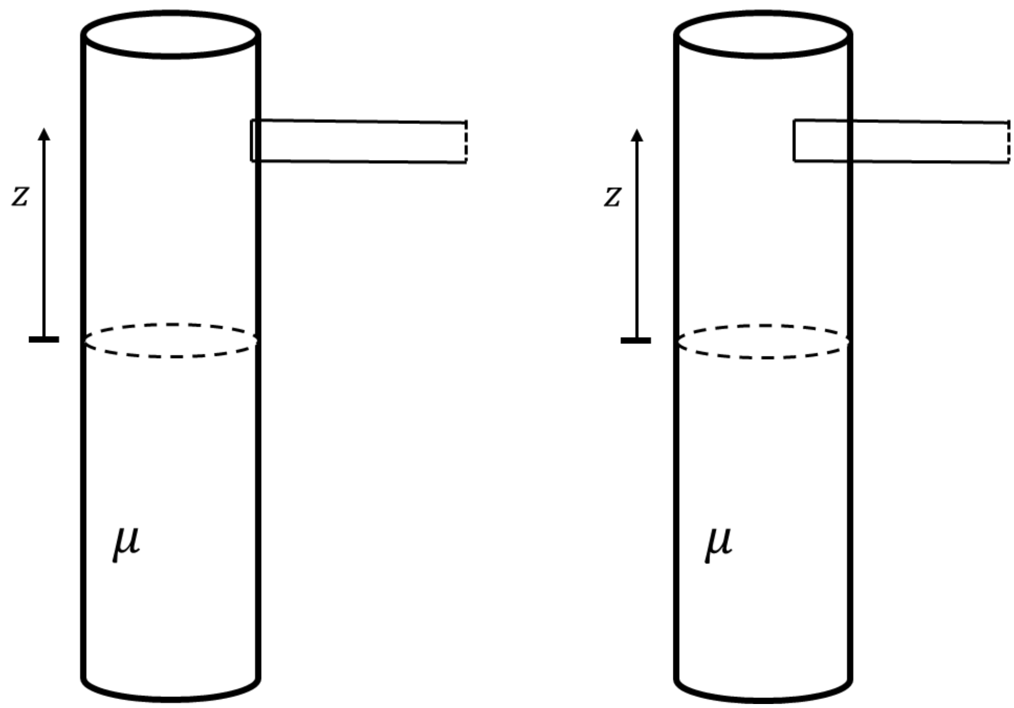
Now, with (13), the inductance is simply, $L = \frac{\mu_0\lambda\pi}{\eta} = \mu_0 \pi R\sqrt{\frac{\mu}{2\eta}} = \mu_0 \pi R\sqrt{\frac{\mu}{ln\mu}}$.

Therefore the asked inductance is:

$$L = \mu_0 \pi R\sqrt{\frac{\mu}{ln\mu}}$$

Appendix 1:Proof that B_z^{in} does not depend on r.

Consider both ampere loops below:



In the first loop we have by amperes law:

$$\int \vec{H} \cdot d\vec{l} = I_{enclosed} = 0 \Leftrightarrow H_z^{in}(R)h + \int_R^\infty \left(H_r^{out}(z+h) - H_r^{out}(z)\right)dr = 0 \ (1)$$

For the second loop, we have again by amperes law:

$$\int \vec{H} \cdot d\vec{l} = 0 \Leftrightarrow H_z^{in}(r)h + \int_r^R \left(H_r^{in}(z+h) - H_r^{in}(z)\right) + \int_R^\infty \left(H_r^{out}(z+h) - H_r^{out}(z)\right)dr = 0 \ (2)$$

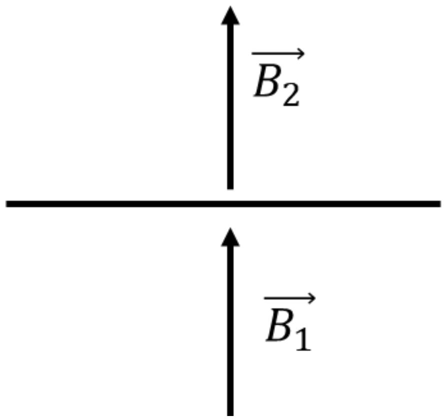
Since at the interface $H_r^{in} = \frac{H_r^{out}}{\mu}$, and $\mu \gg 1$, the second term in the expression above is approximately 0, therefore we obtain:

$$H_z^{in}(r)h = -\int_R^\infty \left(H_r^{out}(z+h) - H_r^{out}(z)\right)dr \ (3)$$

Comparing (1) with (3), we see that $H_z^{in}(r) = H_z^{in}(R)$, therefore H_z^{in} does not depend on r.

Appendix 2:

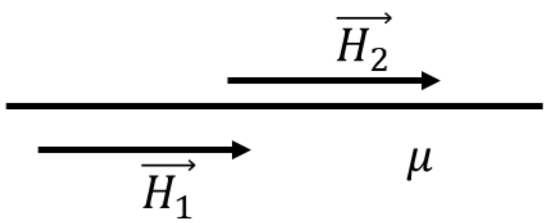
Boundary conditions demonstration:



Let $\overrightarrow{B_1}$ and $\overrightarrow{B_2}$ be the normal components of the magnetic field in medium 1 and 2 respectively. By Gauss law:

$$\int \vec{B} \cdot d\vec{S} = 0 \Leftrightarrow B_2 S - B_1 S = 0 \Leftrightarrow B_1 = B_2$$

Now, let use Amperes law to find the relation between the tangential field components. Now consider the below figure, in which 1 has relative permeability μ and 2 relative permeability 1, and also that $\overrightarrow{H_1}$ and $\overrightarrow{H_2}$ are the tangential H field components(recall that $\vec{B} = \mu\mu_0\vec{H}$).



If the Interface has no free currents(which is the case in the problem), we have that:

$$\int \vec{H} \cdot d\vec{l} = I_{enclosed} = 0 \Leftrightarrow H_2 l - H_1 l = 0 \Leftrightarrow H_1 = H_2$$

Since $\vec{B} = \mu\mu_0\vec{H}$, we have that $\frac{B_1}{\mu} = B_2$