# Physics Cup 2018 - Problem 1. January 28, 2018

Consider infinite lattice of resistors as shown in the figure. Each blue piece of wire between neighbouring nodes has resistance $R_b = 1\,\Omega$, and each red piece of wire between neighbouring nodes has resistance $R_r = 2018\,\Omega$. (Nodes are marked as black circles.) Let the resistance

between the nodes A and B (marked in cyan) be ρ. Find such r and R that
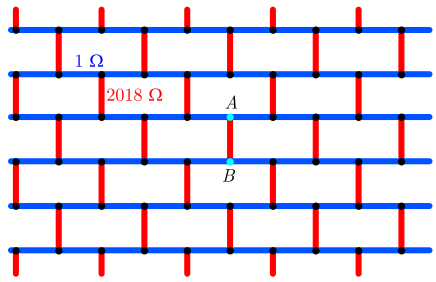
$$r \le \rho \le R \quad \text{and} \quad R/r \le 2.$$

(You need to prove that for the values of r and R suggested by you, these inequalities hold.)

Hints 1–3 (January 7th, 14th, and 21st) Both the lower and upper bound for the resistance can be found by applying idea 27 from the electrical circuits booklet. Both the lower and upper bound can be found as resistances of ladder-type circuits, cf. idea 18 therein. It is possible to obtain both bounds by cutting or short-circuiting all resistors along few horizontal lines.

Hint 4 (January 28, 2018) Be careful when applying the idea 27 from the booklet: if you cut resistances, you'll increase the overall resistance; by short-circuiting something, you'll decrease the resistance. However, if you both cut and short-circuit, the net effect is unknown!

Remark: if you are unable to download the booklet, please let me know (together with the reason why you aren't able to download); I'll send it by e-mail.

Bonus challenge of the last week: Try to find as good as possible bounds with $R/r < 1.22$ (without finding the exact resistance).

Correct solutions submitted during the first four weeks:

name	school	country	pr1 solved
Siddharth Tiwary	Lakshmipat Singhania Academy	India	01 Jan 03:50
Navneel Singhal	ALLEN Kota	India	01 Jan 05:52
Satoshi Yoshida	The University of Tokyo	Japan	01 Jan 13:35
konstantine Gagnidze	Komarovi Tbilisi N199	Georgia	02 Jan 11:25
Prathyush Poduval	Canara PU College	India	02 Jan 16:45
Dylan Toh	NUS High School	Singapore	02 Jan 16:58
Tóbiás Marozsák	Óbudai Árpád Gimnázium	Hungary	03 Jan 18:41
Davit Mdinaradze	Komarovi Tbilisi N199	Georgia	07 Jan 17:16
Gabriel Capelo	Colégio Ari de Sá Cavalcante	Brazil	07 Jan 20:26
Chiosa Ionel-Emilian	International Computer Highschool	Romania	07 Jan 20:52
Peter Elek	DRK Dóczy Gimná zium	Hungary	09 Jan 17:59
Piotr Godlewski	NA (graduated)	Poland	09 Jan 19:03
Dolteanu Stefan	International Computer Highscho	Romania	11 Jan 21:12
Elvinas Ribinskas	University of Cambridge	Lithuania	15 Jan 17:43
Gabriel Domingues	Colégio Etapa	Brazil	19 Jan 19:36
Gabriel Golfetti	Colégio Etapa	Brazil	21 Jan 03:21
Balázs Németh	Budapesti Fazekas Gimnázium	Hungary	21 Jan 08:34
Thomas Bergamaschi	Colegio Etapa Valinhos-Brazil	Brazil	21 Jan 22:15
Michito Ujino	Osaka Seiko Gakuin High School	Japan	28 Jan 07:30