Physics Cup 2018 - Problem 3. March 11, 2018

Find the inductance of a circular loop of wire around an infinite ferromagnetic cylinder of radius r. The cylinder is made from a ferromagnetic material of relative permeability $\mu \gg 1$ (if needed, you can also assume that $\ln \mu \gg 1$); the radius of the loop is slightly larger than r so that it sits tightly around the cylinder.

The hint of 4th March. Keep in mind fact IX-30 from https://www.ioc.ee/~kalda/ipho/formulas.pdf. Also, it might be somewhat useful to read the solutions of http://www.ipho2012.ee/physicscup/problem-no-2/, see http://www.ipho2012.ee/physicscup/problem-no-2/solution/.

Second hint, March 11, 2018. It appears to be convenient to write down differential equations for two unknown functions: (a) axial magnetic field inside the cylinder (the component parallel to the axis of the cylinder), and (b) radial magnetic field outside of the cylinder, near the surface of the cylinder. Although differential equations are involved, the mathematics is actually simple.