**Physics Cup 2018 - Problem 3. March 25, 2018**

Find the inductance of a circular loop of wire around an infinite ferro-magnetic cylinder of radius $r$. The cylinder is made from a ferromagnetic material of relative permeability $\mu \gg 1$ (if needed, you can also assume that $\ln \mu \gg 1$); the radius of the loop is slightly larger than $r$ so that it sits tightly around the cylinder.

*The hints of 4th – 18th March.*
Keep in mind fact IX-30 from https://www.ioc.ee/~kalda/ipho/formulas.pdf. Also, it might be somewhat useful to read the solutions of http://www.ipho2012.ee/physicscup/problem-no-2/, see http://www.ipho2012.ee/physicscup/problem-no-2/solution/.

It appears to be convenient to write down differential equations for two unknown functions: (a) axial magnetic field inside the cylinder (the component parallel to the axis of the cylinder), and (b) radial magnetic field outside of the cylinder, near the surface of the cylinder. Although differential equations are involved, the mathematics is actually simple.

One of the two equations relating axial magnetic field inside the cylinder $B_{ai}$ and radial magnetic field outside of the cylinder $B_{ro}$ to each other can be obtained by writing down Ampère's circulation theorem for the loop shown in the figure. You still need to find a way for resolving the problem of diverging integrals along the radial infinite legs of this loop (you'll obtain a natural logarithm of something).
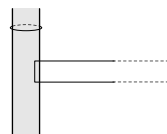
*Fourth hint, March 25, 2018.* The both quantities ($B_{ai}$ and $B_{ro}$) depend very slowly on the coordinate along the axis, let the characteristic scale be denoted by $\lambda$ ($\lambda \gg r$). This characteristic scale is unknown at first, but you'll obtain an expression for it once you solve the two simple differential equations relating $B_{ai}$ and $B_{ro}$ to each other. Now, coming back to the above described logarithmic divergence: it is resolved using the scale $\lambda$ similarly to how such a divergence is resolved when calculating the inductance of a piece of wire of length $l$ and diameter $d$.