Physics Cup 2018 - Problem 5.

Estimate the mean free path of a black sphere of mass m and radius R in vacuum at temperature T. Mean free path here is defined as the travel distance by which the velocity vector of the sphere turns by an angle $\pi/2$. Assume that $kTR \gtrsim hc$, where k and h are the Boltzmann and Planck constants, respectively; c denotes the speed of light. You may find it useful to know that for a random walk when during a time period τ, a step of length a is taken in a random, the average overall displacement during time $t > \tau$ is estimated as $a\sqrt{t/\tau}$.

Fist hint, 6th May 2018. According to the conditions of the problem, the vacuum is described by a single temperature, hence the vacuum must be at thermal equilibrium. This means that it is filled with isotropically propagating electromagnetic radiation, with spectral energy flux density given by Planck's law, and energy flux density given by Stefan-Boltzmann law. (Note that in a moving frame, there would be a Doppler shift and hence a departure from the Planck's and Stefan-Boltzmann laws; in that case, vacuum couldn't be described by its temperature solely.) The black sphere absorbs all the incident photons, and radiates in average as many photons; these radiated photons are in average as energetic as the absorbed ones. Each absorption and each radiation gives rise to a small random change in the velocity of the ball.

Second hint, June 6, 2018. Black sphere is a Brownian particle; its average translational kinetic energy can be found in the same way as that of a molecule. For its velocity to rotate by $\pi/2$, it needs to receive such a momentum from photons which is of the same order of magnitude as its original momentum. The change Δp_x of its x-directional momentum due to all the photons equals to the sum over the contributions from all the absorbed and emitted individual photons, i.e. is a sum over random increments.

Correct solutions submitted:

name	**school**	**country**	**pr 3 solved**
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