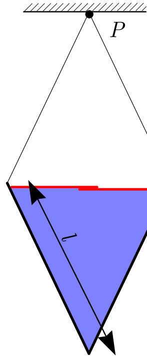
Physics Cup 2018 - Problem 4. May 13, 2018



A V-shaped vessel is made from two plates of width l and length $L \gg l$ which are connected via a frictionless hinge at its bottom. The vessel is fixed to a ceiling using light ropes of length l as shown in figure. The vessel is filled almost up to the rim with water of density ρ, and is subject to the homogeneous gravity field g. The top surface of the water is covered with a weightless thin telescopic plate which cannot be bent, but can be freely extended, and its edges can move frictionlessly up and down along the surfaces of the inclined plates (denoted with a red line in the figure). The purpose of this telescopic plate is to keep the top surface flat during oscillations. The mass of the ropes and plates is negligible. Find the angle between the plates, and the circular frequency of the lowest-frequency mirror-symmetric oscillation mode of this system (evaluate the numerical prefactor of your expression with the precision of four significant digits). Neglect any water motion perpendicular to the plane of the figure.

First hints, 1st – 8th April. In order to understand how water moves during small oscillations, imagine drawing a set of equispaced horisontal lines onto the water. Beneath each line, there is a triangular region filled with water. As the plates move, the shapes of these triangles change, but lines remain horisontal and equispaced. Using this approach one can figure out, how the velocity of the water particles depends on the coordinates.

Apply generalized coordinate approach: express kinetic and potential energies as a function of a small change of the angle between the plates, and of the time derivative of it (other coordinates can be used, as well; these energies can be obtained by integration over the volume of the water). Method is described, for instance, in http://www.ipho2012.ee/physicscup/physics-solvers-3-force-diagrams-or-generalized-coordinates/.

Final hints, 15th and 22nd April. This problem is not so much testing your creativity as your capability of correctly calculating long expressions (although the final answer will be fairly simple). Feel free to use mathematics software, such as Wolfram Alpha to calculate derivatives and handling trigonometry. One way to solve it is sketched in what follows.

To begin with, you'll be needing an expression for the height of the centre of mass h in terms of the half-angle between the plates α, and its first and second derivative $h'(\alpha)$ and $h''(\alpha)$. The answer to the first question can be obtained by solving $h'(\alpha_0) = 0$. For the oscillations period, you'll be needing an expression for the potential energy which can be obtained by using the approximate equality $h \approx h(\alpha_0) + h'(\alpha_0)x + \frac{1}{2}h''(\alpha_0)x^2 = h(\alpha_0) + \frac{1}{2}h''(\alpha_0)x^2$. *As for the kinetic energy,* perhaps the easiest way is to calculate the kinetic energy in the frame of reference of the lowest corner of the system. This frame is moving in the lab frame with a certain velocity $v = A\dot{x}$, where

$x = \alpha - \alpha_0$ and dot denotes time derivative; your task is to find the constant A. Now, you can relate the energy in this moving frame to the energy in the centre-of-mass-frame (CoMF): kinetic energy in any frame equals to the kinetic energy in the CoMF plus the energy of the centre of mass, $Mu^2/2$. We can assume the vertical velocity u of CoMF to be negligibly small so that the energy in the CoMF is the same as the energy in the lab frame. Indeed, $h \approx h(\alpha_0) + h'(\alpha_0)x + \frac{1}{2}h''(\alpha_0)x^2$ so that $u = \dot{h} \approx h''(\alpha_0)x\dot{x}$ [keep in mind that $h'(\alpha_0) = 0$]; we consider only small oscillations so that x remains small and hence $u = \dot{h} \approx h''(\alpha_0)x\dot{x}$ is much smaller than those velocities which don't include the small factor x.
Finally, notice that it is convenient to calculate separately kinetic energy related to horizontal and vertical motion.

For self-checking: the numerical factor for the circular frequency should be between 5 and 6.

All your expressions can be simplified by using the fact that during small oscillations, the angle between plates is almost equal to its equilibrium value α_0.

Correct solutions submitted during the first four weeks:

name	**school**	**country**	**pr 3 solved**
Dylan Toh	NUS High School	Singapore	29 Mar 18:52
Tóbiás Marozsák	Óbudai Árpád Gimnázium	Hungary	1 Apr 17:21
Navneel Singhal	ALLEN Kota	India	2. Apr 12:57
Luciano Rodrigues	Christus	Brazil	15. Apr 13:25
Dolteanu Stefan	Int. Comp. H-sch. Bucharest	Romania	20 Apr 18:58
Muhammad F. Husain	Kharisma Bangsa H-sch.	Indonesia	1 May 13:22
Thomas Bergamaschi	Colegio Etapa	Brazil	12 May 18:22