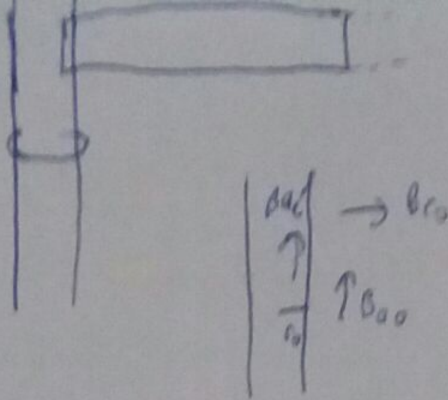
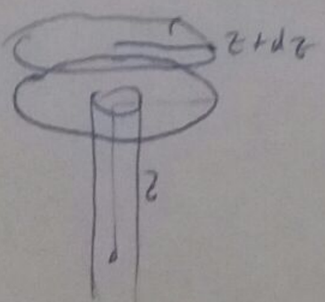


$\oint \vec{H}\cdot d\vec{l} = 0$ for this loop $\Rightarrow$

$$B_{a_i} = -\mu \int_{r_0}^{\infty} \left(\frac{\partial B_{r_0}}{\partial z}\right) dr$$

since $\mu \gg 1$ $\quad B_{a_i} \gg B_{a_0}$

Now if we write $\nabla\cdot\vec{B}=0$ in an approximate form we get:



$$\phi_{(z)} = \phi_{(z+dz)} + 2\pi r\,dz\,B_{r_0}$$

$$-\frac{\partial\phi}{\partial z} = 2\pi r\,B_{r_0}$$

$$B_{r_0} = -\frac{1}{2\pi r}\frac{\partial \phi}{\partial z} \approx -\frac{1}{2\pi r}\frac{\partial \overbrace{B_{a_i}\pi r_0^2}^{\text{flux inside the cylinder}}}{\partial z}$$

$$B_{r_0} \approx -\frac{r_0^2}{2r}\cdot\frac{\partial B_{a_i}}{\partial z}$$

$$B_{a_i} = \frac{\mu r_0^2}{2}\frac{\partial^2 B_{a_i}}{\partial z^2}\int_{r_0}^{\sim} \frac{dr}{r}$$

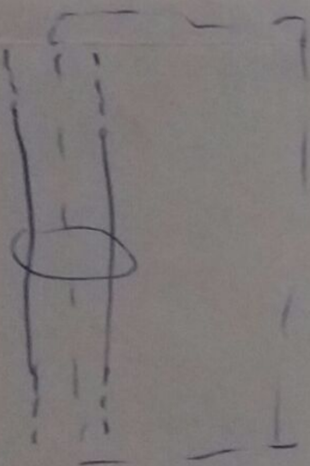
$\rightarrow$ since we approximated the flux we can take the upper limit to an order of λ

$$\lambda = \sqrt{\mu}\,r_0$$

$$B_{a_i} = \frac{\mu r_0^2}{2} \frac{\partial^2 B_{a_i}}{\partial z^2} \ln\sqrt{\frac{\mu}{2}}$$

$$\approx \frac{\mu r_0^2}{4} \frac{\partial^2 B_{a_i}}{\partial z^2} (\ln \mu) \Rightarrow$$

$$B_{a_i} = A e^{-\frac{z}{r_0\sqrt{\frac{\mu}{4}\ln(\mu)}}}$$



$$\oint \vec{H}\cdot d\vec{l} \Rightarrow \quad 2\int_0^\infty B_{a_i}\, dz = \mu\mu_0 I$$

$$A = \frac{\mu\mu_0 I}{2 r_0 \sqrt{\frac{\mu}{4}\ln(\mu)}}$$

$$A = \frac{\mu\mu_0 I}{r_0\sqrt{\mu \ln(\mu)}}$$

$$\boxed{L = \frac{\pi r_0^2 A}{I} = \frac{\mu_0 \mu \pi r_0}{\sqrt{\mu\ln(\mu)}} = \mu_0 \pi r_0 \sqrt{\frac{\mu}{\ln(\mu)}}}$$