Problem 2

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We know the spaceship and the missiles $x(t)$.
t=0 at x=0 spacesip and missiles in the inertia system

1. Spaceship motion

$$x(t) = \frac{c^2}{g}\left(\sqrt{1+\frac{g^2t^2}{c^2}}-1\right) \tag{1}$$

this is a hyperbola

$$\frac{g}{c^2}x^2 + 2x - gt^2 = 0 \tag{2}$$

and the metric

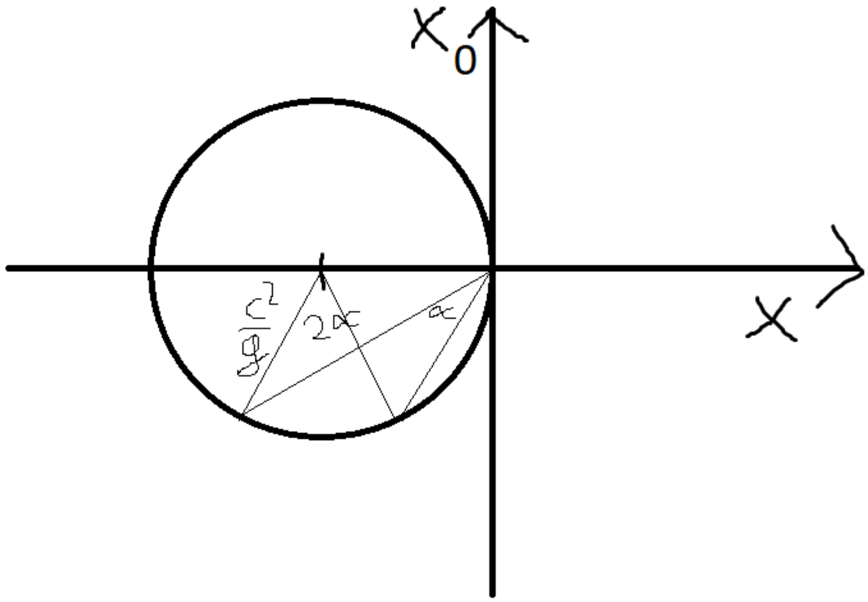
$$c^2 d\tau^2 = -ds^2 = c^2dt^2 - dx^2 \tag{3}$$

τ the spaceship own time
use the first hit and $\mathrm{dx}_0 = icdt$
and the metric

$$-c^2d\tau^2 = ds^2 = dx_0^2 + dx^2 \tag{4}$$

this is the Euklidesian metric and the motion is a circle

$$x^2 + 2x\frac{c^2}{g} + x_0^2 = 0 \tag{5}$$



The curve lenght square $-c\Delta\tau^2$

$$ic\Delta\tau = 2\alpha\frac{c^2}{g} \tag{6}$$

2. Missles motion

the missles motion is a line $x_1 = vt$ and $x_2 = 2vt$

$$\tan\alpha_1 = \frac{x_0}{x_1} = \frac{v}{ic} \tag{7}$$

$$\tan\alpha_2 = \frac{x_0}{x_2} = \frac{2v}{ic} \tag{8}$$

and $\alpha = \alpha_2 - \alpha_1$
use the hiperbolic functions

$$\alpha = i\left(\operatorname{arctanh}\frac{2v}{c} - \operatorname{arctanh}\frac{v}{c}\right) \tag{9}$$

3. The result

$$ic\Delta\tau = 2i\left(\operatorname{arctanh}\frac{2v}{c} - \operatorname{arctanh}\frac{v}{c}\right)\frac{c^2}{g} \tag{10}$$

$$\Delta\tau = 2\left(\operatorname{arctanh}\frac{2v}{c} - \operatorname{arctanh}\frac{v}{c}\right)\frac{c}{g} \tag{11}$$

Let's see the non relativistic situation
if $\left|\frac{v}{c}\right| << 1$

$$\Delta\tau = 2\frac{v}{c}\frac{c}{g} = \frac{2v}{g} \tag{12}$$

this is the exact classical solutions