~~Theorem~~

Q_{max}, the max. heat water can lose to ice $\sim \alpha T \Delta T \sim \alpha T t \lesssim 10 \times 300 \times 10 \; J/kg$

$\sim 10^4 \; J/kg$.

To melt, $Q \sim 10^6 \; J/kg$.

$\therefore$, water remains frozen, and at T_0.

$Q_{net} = 0$

$$\Rightarrow 0 = \lambda \Delta m + \left(1 - \frac{1}{n}\right) m \alpha \int_{T_0 - t}^{T_0} T \, dT + \frac{1}{n} m \alpha \int_{T_0 - t}^{T_f} T \, dT$$

$$\Rightarrow \frac{n \lambda \Delta m}{m \alpha} + \frac{(n-1)}{2}\left(T_0^2 - (T_0 - t)^2\right) + \frac{1}{2}\left(T_f^2 - (T_0 - t)^2\right) = 0$$

$[\Delta m = \text{change in } H_2O \text{ mass}]$

$$\Rightarrow \frac{n \lambda \Delta m}{T_0 m \alpha} + (n-1) t \left(1 - \frac{t}{2T_0}\right) + \frac{1}{2T_0}(T_f - T_0 + t)(T_f + T_0 - t) = 0 \quad \text{(i)}$$

Reversible $\Rightarrow \Delta S = 0$

$$\Rightarrow \int \frac{\lambda \, dm}{T_0} + \left(1 - \frac{1}{n}\right) m \alpha \int_{T_0 - t}^{T_0} \frac{T}{T} dT + \frac{1}{n} m \alpha (T_f - T_0 + t) = 0$$

$$\Rightarrow \frac{n \lambda \Delta m}{T_0 m \alpha} + (n-1) t + (T_f - T_0 + t) = 0 \quad \text{(ii)}$$

(ii) − (i)

$$\Rightarrow (n-1)\frac{t^2}{2T_0} + (T_f - T_0 + t)\left(1 - \frac{T_f}{2T_0} - \frac{1}{2} + \frac{t}{2T_0}\right)$$

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$\frac{t}{T_0} = t'$, $\frac{T_f}{T_0} = T_f'$

$\therefore \frac{(n-1)}{2} t'^2 + \frac{(t' + T_f' - 1)}{2} (t' - (T_f' - 1)) = 0$

$\Rightarrow (n-1) t'^2 + t'^2 - (T_f' - 1)^2 = 0$

$\Rightarrow n t'^2 = (T_f' - 1)^2$

$\Rightarrow 1 \pm \sqrt{n}\, t' = T_f'$

$\therefore$ It is possible to raise the temperature of the ~~fl~~ pan to atmost

$T_0 + \sqrt{n}\, t$, or chill it down to $\boxed{(T_0 - \sqrt{n}\, t)}$

Our discussion didn't rely on the specifics of the process, so regardless of the number / configuration of engines and refrigerators used, the limits are unsurpassable.