Problem No 2

Cooling of n-th part of the ice can be considered as two processes:

1) Heat engine heats m$\frac{n-1}{n}$ of ice, cools m of water and produces work A
2) Refrigerator uses work A to cool m$\frac{1}{n}$ and heat water with ice

Final temperature T_{min} will be the lowest if we use the Carnot cycles.
Because of $\alpha T_0 t << \lambda$ only small amount of water will be frozen but all the ice will be at temperature T_0 in the end. In the Carnot cycle

$$\text{dA}=\text{d}Q_{ice}(\frac{T_{water}}{T_{ice}}-1)=\text{d}Q_{ice}\frac{t}{T_0-t}=-\frac{n-1}{n}\alpha t dt$$

Thus

$$A = m\alpha\frac{n-1}{n}\frac{t^2}{2} \tag{1}$$

In the second process

$$dA = dQ_{n-th}(\frac{T_{mix}}{T_{n-th}}-1) = dQ_{n-th}\frac{t}{T_0-t} = -\frac{m}{n}\alpha t dt$$

where T_{mix} is the temperature of mixture of water and ice and T_{n-th} is the temperature of n-th part of ice.
This gives

$$A = \frac{m\alpha}{n}\frac{t_{final}^2-t^2}{2} \tag{2}$$

Solution of the system of equations (1) and (2) is:

$$t_{final} = t\sqrt{n} \tag{3}$$

And finally:

$$T_{min} = T_0 - t\sqrt{n} \tag{4}$$