Problem 5

Physics Cup

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Since all surfaces in the diffraction grating have sides $S = 20\lambda$ for all wavelengths diffraction effects will be equal for all squares in the grating. A rectangular aperture gives an amplitude

$$A \propto sinc(\frac{\pi}{\lambda}Scos(\alpha))sinc(\frac{\pi}{\lambda}Scos(\beta)) \tag{1}$$

Where $cos(\alpha)$ and $coa(\beta)$ are the directional cosines of the outgoing rays.

In order to find the maxima of the diffraction pattern we can use the fact that a smaller angle between the outgoing rays and the normal to the surfaces results in higher intensity when $cos(\alpha) < \frac{\lambda}{2S}$ and $cos(\beta) < \frac{\lambda}{2S}$. Therefore, as we will find that the maxima of the interference pattern are extremely sharp peaks all of equal size the global maxima will be the peak were rays are closest to the surface normal.

Instead of using coordinates x, y to describe the grating we introduce coordinates n, m such that $Sn = x$, $Sm = y$ where n and m are whole numbers and S is the width of one reflective square in the grating ($S = 20 * \lambda$). Each pair (n, m) corresponds to one square in the grating and $-250 < n, m < 250$. Thus we can rewrite the equation for z to

$$z = An + Bm \tag{2}$$

Where:
$A = 20\lambda_0$
$B = \lambda_0$

When the incident light ray hits the grating it will hit different surfaces at different phases and thus we can expect each square to emit light at a different phase ϕ_0 depending on $z(n, m)$. If we set $\phi_0 = 0$ at $(n, m) = (0, 0)$ then $\phi_0 = -\frac{2\pi}{\lambda}z = -\frac{2\pi}{\lambda}(An + Bm)$

When light rays are emitted from a point $\vec{r}$ in some direction parallel to the unit vector $\vec{u}$ the phase shift between this light and light emitted in the same direction from the origin is

$$\phi_1 = -\frac{2\pi}{\lambda}\vec{u} \cdot \vec{r} \tag{3}$$

or equivalently

$$\phi_1 = -\frac{2\pi}{\lambda}(xcos(\alpha) + ycos(\beta) + zcos(\gamma)) \tag{4}$$

In terms of n, m this yields

$$\phi_1 = -\frac{2\pi}{\lambda}((Scos(\alpha) + Acos(\gamma))n + (Scos(\beta) + Bcos(\gamma))m) \tag{5}$$

Adding up these two phase shifts we get

$$\phi = -\frac{2\pi}{\lambda}((Scos(\alpha) + A(1 + cos(\gamma)))n + (Scos(\beta) + B(1 + cos(\gamma)))m) \tag{6}$$

The interference maxima correspond to the points where ϕ is some multiple of 2π for all n and m. Thus

$$Scos(\alpha) + A(1 + cos(\gamma)) = i\lambda \tag{7}$$

$$Scos(\beta) + B(1 + cos(\gamma)) = j\lambda \tag{8}$$

Where i, j are two whole numbers.

Finally we can find the global maxima by choosing i, j such that $cos(\alpha)$ and $cos(\beta)$ are minimized. At this point γ will be so small that we can approximate $cos(\gamma) = 1$. Thus:

$$cos(\alpha) = \frac{1}{S}(i\lambda - 2A) \tag{9}$$

$$cos(\beta) = \frac{1}{S}(j\lambda - 2B) \tag{10}$$

This results in the following pattern when the distance to the screen is 50 cm

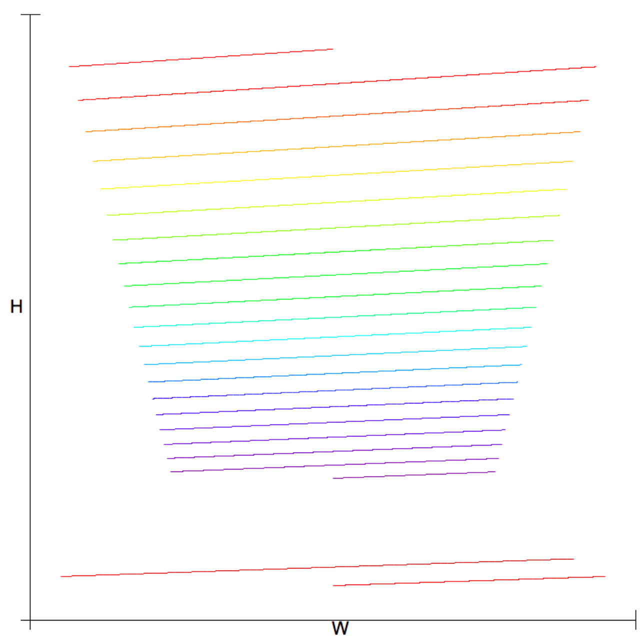

Figure 1: Diffraction pattern

Where:
$H = 40$mm
$W = 40$mm