Physics Cup – TalTech 2019 – Problem 1. December 9, 2018

When a body moves in liquid, the motion of the body puts the liquid into motion, too. The motion of liquid contributes to the total kinetic energy of the system, and hence, leads to an increased *effective mass* of the body. The difference of the effective mass and the actual mass of the body is referred to as the *added mass*. The added mass depends on the size and shape of the body.

Consider a certain metallic body of volume V and polarizability α along its symmetry axis x (i.e. homogeneous externally applied electric field $\vec{E}$ induces total dipole moment $\vec{p} = \alpha\vec{E}$ on this body). Additionally, the body shape is such that if it were made from a homogeneous dielectric material and put into homogeneous electric field, the electric field inside the body would be also homogeneous. Find the added mass of this body when it starts moving translationally, parallel to the x-axis, in an incompressible initially motionless liquid of density ρ. The viscosity of the liquid is negligibly small. Express the answer in terms of V, ρ, α, and physical constants.

Hint. Initially vortex-free inviscid fluid remains free of vortices, i.e. $\oint \vec{v}\mathrm{d}\vec{r} = 0$ for any integration contour inside the liquid, where $\vec{v} \equiv \vec{v}(\vec{r}, t)$ is the fluid velocity at position $\vec{r}$; t denotes a fixed moment of time.