Physics Cup – TalTech 2019 – Problem 1. January 13, 2019

When a body moves in liquid, the motion of the body puts the liquid into motion, too. The motion of liquid contributes to the total kinetic energy of the system, and hence, leads to an increased *effective mass* of the body. The difference of the effective mass and the actual mass of the body is referred to as the *added mass*. The added mass depends on the size and shape of the body.

Consider a certain metallic body of volume V and polarizability α along its symmetry axis x (i.e. homogeneous externally applied electric field $\vec{E}$ induces total dipole moment $\vec{p} = \alpha\vec{E}$ on this body). Additionally, the body shape is such that if it were made from a homogeneous dielectric material and put into homogeneous electric field, the electric field inside the body would be also homogeneous. Find the added mass of this body when it starts moving translationally, parallel to the x-axis, in an incompressible initially motionless liquid of density ρ. The viscosity of the liquid is negligibly small. Express the answer in terms of V, ρ, α, and physical constants.

Hint. Initially vortex-free inviscid fluid remains free of vortices, i.e. $\oint \vec{v}\mathrm{d}\vec{r} = 0$ for any integration contour inside the liquid, where $\vec{v} \equiv \vec{v}(\vec{r}, t)$ is the fluid velocity at position $\vec{r}$; t denotes a fixed moment of time.

Hint 1 (16th Dec. 2018). Build an electric field with field lines matching the streamlines of liquid around the body by considering a linear combination of an homogeneous electric field, and the field created by an electrically polarized body.

Hint 2 (23rd Dec. 2018). In the case of the electric field from the previous hint, relate the energy of the field to the electrostatic interaction energy between the polarization charges and the external field on the one hand, and to the kinetic energy of the flow around the body on the other hand.

Hint 3 (30th Dec. 2018). Let us be more specific with the previous hints. Regarding the first hint, you need to show that the equations defining the velocity field and the electric field are the same. So, if the boundary conditions for the field at the surface of the body are identical in both cases, due to the uniqueness of a correctly posed problem, the solutions would be identical, too. Your task is to figure out such an electrostatic configuration for which the boundary conditions are the same as in the case of fluid flow.

Regarding the second hint, what you are suggested to do is expressing the electrostatic interaction energy as: (a) an integral of the energy density of the field ; (b) potential energy of the electric dipole (induced on the body) in the homogeneous external electric field. (NB! be careful and do not lose the factor $\frac{1}{2}$). Since these two things must be equal, you'll obtain an expression for the integral of the energy density.

Hint 4 (6th January 2019. Let us sum up what you need to do (summary of all the hints + small additional recommendations).

Step 1. Show that electrostatic field and vortex-free fluid flow are described by the same set of equations in the bulk.

Step 2. Build such an electrostatic field which has the same shape of field lines as the velocity field. You can match the field lines either in a co-moving frame where there is a stationary flow around the body, or in the resting lab frame; in the latter case, the field lines correspond an instantaneous snapshot of the flow. To build such a field, you need to use the fact that there is only one solution to a correctly posed boundary value problem, i.e. the same boundary conditions need to be satisfied at the surface of the body, and at infinity, for the both cases. (Conditions at infinity depend on whether you use a co-moving frame or a resting frame.) To build the field of required properties, consider a linear combination of two electrostatic fields: homogeneous field, and the field created by the electrically polarized body.

Step 3. Express the electrostatic interaction energy of the polarized body and the external field in terms of the external field strength and the induced dipole moment.

Step 4. Express the electrostatic interaction energy of the polarized body and the external field as an integral of the energy density of the electric field.

Step 5. Express the kinetic energy of the fluid motion as an integral over space.

Step 6. Hence, deduce the added mass.

Bonus hint of 13th January 2019. Since this is your last chance with Problem 1, the last hint is about how to increase the probability of catching and fixing smaller mistakes (typos) by yourself. Check the veracity of your answer: check if it has a correct behavior at the limit of a needle-like object (large polarizability, small added mass). You can also search in internet (or calculate by yourself) the added mass and polarizability of a sphere, and check if these are in agreement with your formula .

By the end of the fifth week, there were 375 registered participants from 54 countries; among them there were 178 high school students, and 197 university students. During the first three weeks, in total 76 solutions were submitted, out of which 21 were correct. For the university students, there is still a chance of getting the speed bonus!

NB! This time, you are given one extra week to submit your solution, until the midnight of Sunday — 19th January (GMT).

Correct solutions submitted by January 13, 2019:

Name	Uni/PreUni	country	subm. time (GMT)
Yunus Emre Parmaksız	PreUni	Turkey	10 Dec. 2018, 9:36
Thomas Foster	Oxford	UK	10 Dec. 2018, 16:36
Ionel-Emilian Chiosa	PreUni	Romania	11 Dec. 2018, 18:35
Gabriel Trigo	PreUni	Brazil	12 Dec. 2018, 21:03
Oliver Lindström	PreUni	Sweden	13 Dec. 2018, 1:00
Johanes Suhardjo	HKUST	Indonesia	16 Dec. 2018, 7:44
Ivan Ridkokasha	Шевченко	Ukraine	16 Dec. 2018, 21:25
Felix Christensen	Oxford	Germany	19 Dec. 2018, 0:19
Tùng Trần	PreUni	Vietnam	19 Dec. 2018, 17:37
Maria Amcique	PreUni	France	19 Dec. 2018, 21:49
Mateusz Kapusta	PreUni	Poland	20 Dec. 2018, 18:56
Vladislav Polyakov	PreUni	Russia	21 Dec. 2018, 16:36
Farhan Husain	PreUni	Indonesia	24 Dec. 2018 7:00
Samarth Hawaldar	Indian Inst of Sci	India	25 Dec. 2018, 22:02
Mustafa Tugtekin	PreUni	Turkey	25 Dec. 2018, 22:02
Oliwier Urbański	PreUni	Poland	29 Dec. 2018, 11:31
Morteza Mudrick	PreUni	Indonesia	31 Dec. 2018, 3:23
Roberto Marín Delgado	PreUni	Costa Rica	02 Jan. 2019, 14:31
Stefan Dolteanu	PreUni	Romania	06 Jan. 2019, 12:23
Eduard Burlacu	PreUni	Romania	12 Jan. 2019, 23:04
Gusti P S G Atmaja	PreUni	Indonesia	13 Jan. 2019, 4:56