Physics Cup – TalTech 2019 – Problem 2. January 13, 2019

Typically, the Debye model describes well the heat capacity C_V of crystals: for absolute temperatures T much smaller than the so-called Debye temperature T_D, $C_V \propto T^3$; at the opposite limit of $T \gg T_D$, all the crystal lattice oscillations are thermally unlocked and hence, $C_V \approx 3Nk_B$, where N is the total number of atoms.

Surprisingly, the heat capacity of ice behaves in a totally different manner: with a very good precision and over a wide range of temperatures (from the melting point T_0 down to ca $100\,\mathrm{K}$), its specific heat c_V is proportional to the absolute temperature. In what follows, you may assume that for ice, $c_V = \alpha T$, where $\alpha \approx 7.51\,\mathrm{J{\cdot}kg^{-1}K^{-2}}$, and the latent heat of melting $\lambda = 334\,\mathrm{kJ/kg}$.

Consider an isolated system consisting of equal masses m of water at temperature $T_0 = 273.15\,\mathrm{K}$, and ice. The ice is at a slightly lower temperature $T_0 - t$, where t is of the order of few kelvins. This isolated system includes also ideal reversible heat engines of negligible heat capacity. One heat engine is used to produce mechanical energy which is consumed by another heat engine which is operated as a refrigerator. What is the lowest temperature T which can be given to a n-th part of the ice (of mass m/n) if $n \gg 1$?