Physics Cup – TalTech 2019 – Problem 3. March 3, 2019

An elastic ball of negligible size is dropped vertically onto a frictionless inclined plane which makes an angle α with the horizontal. Initially the distance between the plane and the ball is d and the speed of the ball is zero. The trajectory of the bouncing ball consists of parabolic arcs. Show that the foci of these parabolic arcs lie on a well-known shape, and give the parameter(s) defining this shape.

Assume that collisions are perfectly elastic and that air drag is negligible.

The hint of 17th Feb. 2019. There are many ways to reach the final answer, but the shortest one uses geometrical approach. Try to prove and use the following lemma: the directrix of all the parabolas is the same horizontal straight line, passing through the initial position of the ball.

The hint of 24th Feb. 2019. Using a slanted coordinate system one can find that all the parabolic arcs are touching the line which is parallel with the inclined plane and passes through the initial position of the ball. Try to use this line and the directrix mentioned in the first hint.

The hint of 3rd Mar. 2019. Besides the first two hints, you may need to use the reflective property of the parabola, i.e. that the reflection of a light ray starting from the focus of a parabolic mirror is a line parallel with the symmetry axis of the parabola.

By the end of the third week of the third problem, there were 395 registered participants from 54 countries; among them there were 196 high school students, and 199 university students. During the first week, in total 65 solutions were submitted, out of which 46 were correct.

Correct solutions submitted by March 3, 2019:

Name	Uni/PreUni	country	subm. time (GMT)
Siddharth Tiwary	Uni	India	10 Feb., 14:38
Ionel-Emilian Chiosa	PreUni	Romania	10 Feb., 15:55
Thomas Foster	Uni	UK	10 Feb., 16:16
Marvin Janini	PreUni	Brazil	10 Feb., 16:55
Johanes Suhardjo	Uni	Indonesia	10 Feb., 17:15
Tùng Trân Xuân	PreUni	Vietnam	10 Feb., 17:26
Damiano Tietto	Uni	Italy	10 Feb., 17:49
Domagoj Perkovič	PreUni	Croatia	10 Feb., 18:30
Ivan Ridkokasha	Uni	Ukraine	10 Feb., 18:57
Eduard Burlacu	PreUni	Romania	10 Feb., 19:44
Stefan Doteanu	PreUni	Romania	10 Feb., 21:26
Marcell Szakály	Uni	UK/Hungary	11 Feb., 00:13
Gabriel Trigo	PreUni	Brazil	11 Feb., 02:37
Roberto Marín Delgado	PreUni	Costa Rica	11 Feb., 05:17
Oliver Lindström	PreUni	Sweden	11 Feb., 10:25
Vladislav Polyakov	PreUni	Russia	11 Feb., 11:16
Ivander Jonathan M. W.	PreUni	Indonesia	11 Feb., 16:12
Zāhārāchesku Mihai	PreUni	Romania	11 Feb., 17:06
Masuk Ridwan	PreUni	Bangladesh	12 Feb., 11:51
Oliwier Urbański	PreUni	Poland	12 Feb., 20:51
Domonkos Svastics	PreUni	Hungary	12 Feb., 21:05
Bakir Devedzic	PreUni	Bosnia & Hercegovina	13 Feb., 03:18
Siam Absur Khan	PreUni	Bangladesh	13 Feb., 11:22
Aaditya Mishra	Uni	India	16 Feb., 07:16
Timothy Ho	PreUni	Singapore	16 Feb., 09:32
Erik Swanson	PreUni	USA	16 Feb., 10:46
Eduardo Bardal Slikta	PreUni	Brazil	17 Feb., 06:06
Roshan Kaundinya	Uni	India	17 Feb., 09:23
Petra Brčić	Uni	Croatia	17 Feb., 10:50
Domantas Burba	PreUni	Lithuania	17 Feb., 11:41
Vaibhav Raj	PreUni	India	17 Feb., 13:56
Yasin Sönmez and Batuhan Keskin	Uni	Turkey	17 Feb., 18:01
Ferreira Rodrigues	PreUni	Brazil	20 Feb., 12:20
Mateusz Kapusta	PreUni	Poland	21 Feb., 20:03
Stephen Catsamas	PreUni	Australia	23 Feb., 17:43
Morteza Mudrick	PreUni	Indonesia	24 Feb., 12:56
Bartłomiej Sikorski	PreUni	Poland	24 Feb., 18:13
Jaeyong Lee	PreUni	Korea	24 Feb., 19:24
Dhyaan S. Nayak	PreUni	India	25 Feb., 09:06
Richard Luhtaru	PreUni	Estonia	26 Feb., 17:19
Ayush Anand	PreUni	India	28 Feb., 05:57
Kaviraj Prithvi	PreUni	India	1 Mar., 13:59
Jianzhi Wang	PreUni	Singapore	2 Mar., 04:02
Muhamed Sokolović	PreUni	Bosnia & Hercegovina	2 Mar., 10:11
Felix Bekir Christensen	Uni	UK/Germany	2 Mar., 11:17
Esha Manideep Dinne	PreUni	India	2 Mar., 12:19
Tádé Marozsák	PreUni	Hungary	2 Mar., 18:42