Physics Cup – TalTech 2019 – Problem 4. March 10, 2019

Consider an infinite square grid of resistors. Let us introduce coordinates x and y so that all the nodes are at integer coordinates (n, m), with $n, m \in \mathbb{Z}$. For this grid of resistors, all the horizontal resistors, i.e. the resistors between node pairs $[(n, m), (n + 1, m)]$, have the same resistance R; all the vertical resistors, i.e. the resistors between node pairs $[(n, m), (n, m + 1)]$ have the same resistance r. It appears that for such a grid, the effective resistance R_{nn} between the nodes $(0, 0)$ and (n, n) equals to

$$R_{nn} = \frac{2\sqrt{Rr}}{\pi} \sum_{k=1}^{n} \frac{1}{2k-1};$$

this formula can be used in your solution. By how much will change the effective resistance between the nodes $(0, 0)$ and $(1, 1)$ when the nodes (n, n) and $(n + 1, n + 1)$ are connected with a piece of wire of negligibly small resistance? In other words, determine $R'_{11} - R_{11}$, where R'_{11} is the new effective resistance between the nodes $(0, 0)$ and $(1, 1)$ after short-circuiting the nodes (n, n) and $(n + 1, n + 1)$. Assume that $n > 1$.