Physics Cup – TalTech 2019 – Problem 5. May 12, 2019

Consider a check-board-like reflective diffraction grating a cross-section of which is shown in the figure below. The height of the reflecting surface above a reference plane is given by

$$z = 20\lambda_0 \left\lfloor \frac{x}{20\lambda_0} \right\rfloor + \lambda_0 \left\lfloor \frac{y}{20\lambda_0} \right\rfloor,$$

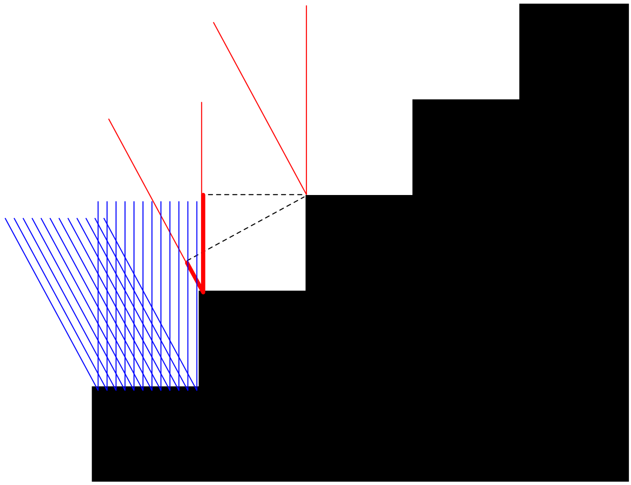
where $\lambda_0 = 500\,\mathrm{nm}$, $\lfloor a \rfloor$ denotes the floor function (real number a is rounded down to the nearest integer), and $|x|, |y| < 5000\lambda_0$ (thus, the size of the grating is $5\,\mathrm{mm} \times 5\,\mathrm{mm}$). The side surfaces (vertical edges in the figure below) of the "stairs" are black and absorb all the incident light. A parallel beam of white light containing all the wavelengths from $\lambda_1 = 400\,\mathrm{nm}$ to $\lambda_2 = 700\,\mathrm{nm}$ propagates parallel to the z-axis and falls onto the grating. The reflected beam is focused with a lens onto a screen perpendicular to the beam at $z = 50\,\mathrm{cm}$. Sketch qualitatively the pattern which can be seen on the screen and show the approximate dimensions of the pattern. For each wavelength, mark only the main diffraction maximum (the brightest spot). Your answer should be a set of points at $x-y$ plane (at $z = 50\,\mathrm{cm}$) showing the brightly illuminated region on the screen.

NB! The score for this problem is split into two parts: a sketch which captures the most important features of what can be on the screen will give a score of 0.5, and a fully correct sketch - the remaining 0.5 pts. The speed bonus and the penalties for submitting wrong solutions will be applied separately to the both sub-scores.

The hint of 28th April First about how your answer should look like. For each wavelength, the brightest spot is basically a dot on the screen. As wavelength changes, the dot on the screen moves. So, the answer should be a line/curve or a set of curves. Second, in order to have a constructive interference, there should be a constructive interference for the beams adding up in the x-direction and in the y-direction (since any destructive superposition will negate anything there was before). Third, when figuring out which of the spots is the brightest one, it might be helpful to recall the intensity distribution over angles for a diffraction grating.

The hint of 5th May Keeping in mind the previous hints, this problem, although involving 3D rays and 2D diffraction grating, can reduced to solving 1D grating problems with 2D ray patterns. So, you can start considering the effect of one set of stairs steps in the stripe $0 \leq x \leq 20\lambda_0$, the side view of which looks almost exactly like what is shown in the figure above, except that the stair step height equals to the stair step width. Obviously there are many wavelengths λ_n for which the main maximum is observed for a reflected beam propagating strictly parallel to the surface normal; this happens for $n\lambda_n = 20\lambda_0$, where n is an integer. These 'bright spots" are as bright as it can be as all the contributing rays (hitting the reflecting surface at different parts of a stair step tread) are exactly at the same phase. Next let us consider a certain wavelength $\lambda_n' = \lambda_n + \delta\lambda$ which is a little bit larger than λ_n. Due to a small wavelength increment, the neighbouring stair steps contribute now slightly off-phase for the strictly normal back-propagation which means that the direction of the in-phase-propagation (the direction of the brightest spot) becomes now slightly tilted. It also means that the brightest spot is now slightly less bright as different parts of a single stair step tread are no longer exactly at the same phase. The same approach can be used for another set of stairs steps, this time forming a x-directional stripe with $0 \leq y \leq 20\lambda_0$. Finally, the hint of 28th April can be used to bring the obtained results together.

The hint of 12th May To make the previous hint more clear, here is the sketch illustrating which beams you need to consider. To decide, at which directions bright spots can be observed for the given wavelength, consider the interference of the pair of beams shown as red lines (optical path difference is shown as a bold line; this drawing corresponds to a $z-x$-plane). To decide, which of the bright spots for the given wavelength are brighter and which as dimmer, consider the interference of the beams shown as blue lines.



By the end of the fourth week of the fifth problem, there were 405 registered participants from 55 countries; among them there were 204 high school students, and 201 university students. During the first three weeks, in total 13 solutions of the fifth problem were submitted, out of which 4 were correct, and 1 more qualifies as the one capturing the most important features.

Correct solutions submitted by 12th May 2019:

Solutions capturing the most important features of the pattern which can be seen on the screen (but possibly incorrect in details).

Name	country	Uni/PreUni	subm. date/time (GMT)
Thomas Foster	UK	Oxford	14 Apr. 2019 17:22
Johanes Suhardjo	Indonesia	HKUST	14 Apr. 2019 18:34
Oliver Lindström	Sweden	PreUni	16 Apr. 2019 10:19
Oliwier Urbański	Poland	PreUni	17 Apr. 2019 10:35
Tùng Trần Xuân	Vietnam	PreUni	29 Apr. 2019 19:37

Fully correct solutions.

Name	country	Uni/PreUni	subm. date/time (GMT)
Oliver Lindström	Sweden	PreUni	16 Apr. 2019 10:19
Johanes Suhardjo	Indonesia	HKUST	20 Apr. 2019 14:27
Oliwier Urbański	Poland	PreUni	23 Apr. 2019 13:13
Thomas Foster	UK	Oxford	24 Apr. 2019 19:54