

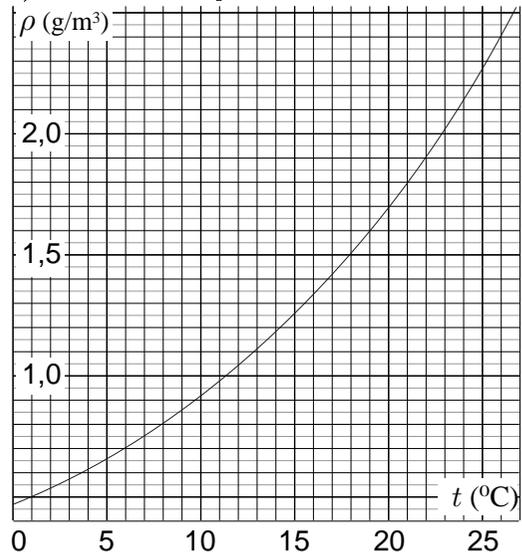
1. Drying (12 p)

According to the wide-spread belief, it is useful to keep window open when drying laundry even if the relative humidity outside is 100%, because the temperature of the incoming air rises and thereby the relative humidity drops. Let us analyse, do these arguments hold, when heating is switched off.

Suppose that inside a room, the volume of air $V_1 = 20\text{ m}^3$ from inside at the temperature $t_1 = 25\text{ C}^\circ$ is mixed with the volume of air $V_2 = 10\text{ m}^3$ from outside at the temperature $t_2 = 1\text{ C}^\circ$. The specific heat of the air (by fixed pressure) $c_p = 1005\text{ J}/(\text{kg}\cdot\text{K})$ can be assumed to be constant for the given temperature range; the heat exchange with the medium can be neglected. For the time being, you may neglect the possibility of (partial) condensation of the vapour.

1) Prove that the total volume of the air will not change i.e. that the volume of the mixed air $V = V_1 + V_2$.

2) What is the temperature of the mixed air T ?



3) The graph below shows the dependence of the saturated vapour density for water as a function of temperature. Before mixing, both the interior and exterior air had relative humidity of $r_0 = 100\%$. What is the relative humidity r of the mixed air (if it happens to increase then assume that an oversaturated vapour with

$r > 100\%$ is formed)?

4) If you happened to obtain $r > 100\%$ then the oversaturated vapour breaks down into a fog which contains tiny water droplets. In that case, what is the mass m of the condensed water (i.e. the total mass of the water droplets)? Air density $\rho_0 = 1,189\text{ Kg}/\text{m}^3$; latent heat of vaporization for water $q = 2500\text{ kJ}/\text{kg}$.

2. Photography (7 p)

By taking measurements from the photo at the end of the page, determine the diameter of the camera lens used for this photo. You can assume that images created by this camera lens are identical to ones created by ideal thin lens of matching focal length and diameter.

3. Sucking (7 p)

Let a large vessel be filled with an incompressible dielectric liquid of density of mass density ρ_m (the relative dielectric permeability $\epsilon \approx 1$). This liquid carries homogeneous volume charge of density ρ_e which is so small that the electric field E_0 created by it is negligible: $E_0\rho_e \ll g\rho_m$, where g is the free fall acceleration. Surface tension can be also neglected. All the heights will be measured from the unperturbed height of the liquid surface. A point charge $-q$ of opposite sign is brought to the height H , due to which a kink is formed on the liquid surface.

1) Determine the height of the kink a .

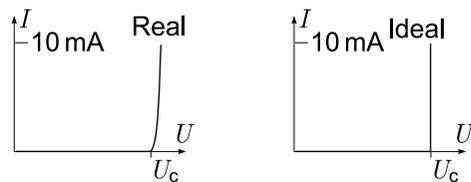
2) If the height of the charge is slowly decreased, at which height h the liquid will start flowing to the point charge?

4. Electric experiment (12 p)

Find the capacitance of an unknown capacitor and estimate the experimental uncertainty. Equipment: red light emitting diode (LED), three resistors one of resistance $R_1 = 1.5\text{ k}\Omega$ one of resistance $R_2 = 6.2\text{ k}\Omega$, and one of unknown resistance; a battery of unknown electromotive force (internal resistance is smaller than 500Ω), wires, timer, unknown capacitor.

Remarks: below is provided a typical $V - I$ curve of a LED; during this experiment, the $V - I$ curve of the LED can be approximated with that of an ideal diode, cf. graph. The value of the opening voltage U_c of the LED is not known. When there is a non-zero current

through the LED, it emits light.



If a capacitor of capacitance C and resistor of resistance R are connected in series to an electromotive force E then the capacitors voltage will approach exponentially its asymptotic value: $U = E \pm U_0 e^{-t/RC}$.



5. Empty bag (12 p)

A cylindrical bag is made from a freely deformable fabric, impermeable for air, which has surface mass density σ ; its perimeter L is much less than its length l . If this bag is filled with air, it resembles a sausage. The bag is laid on a horizontal smooth floor (coefficient of friction $\mu = 0$). The excess pressure inside the bag is p , the free fall acceleration — g . The density of air is negligible.

1) Find the width of the contact surface between the bag and the floor c .

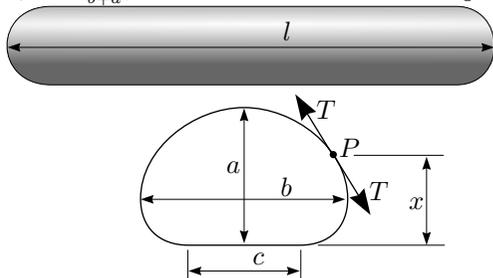
2) Prove that the tension in the bag's fabric $T = \alpha x + \beta$, where x is the height of the given point P above the floor, and find the coefficient α . *Remark:* fabric's tension is (loosely speaking) the force per unit length. Actually, fabric's tension is not as simple concept as a rope's tension, because different force directions are possible; here, however, we neglect the force component along the "sausage's" axis. Thus, we can describe the fabric's tension here with a single number, the force per unit length T acting across an horizontal cut of the "sausage".

Hint: consider the force balance for a small piece of fabric (using the vertical cut shown in figure).

3) Let the highest point of the bag be at height a from the floor. What is the tension T_1 at this highest point? Express your answer in terms of a , σ and p (or α , if you were unable to answer the previous question).

Hint: consider the force balance between two halves of the bag.

4) Assuming that $p \gg \sigma g$, determine the quantity $\varepsilon = \frac{b-a}{b+a}$, where b is the width of the bag.



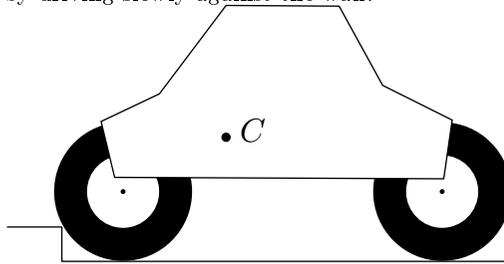
6. Car (7 p)

A car attempts driving over a road barrier, starting from rest, as shown in figure. The diameter of its wheel $d = 1$ m, coefficient of friction between the wheels and ground $\mu = 1$. The sketch is drawn using correct proportions, point C marks the position of the car's centre of mass. Determine the maximal height of those road barriers which can be driven over with such a car, assuming that there is no drag in the non-driving axis (i.e. wheels rotate without friction), and the driving wheels are

1) front wheels;

2) rear axis.

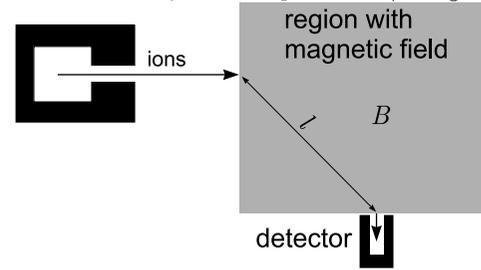
3) Suppose that the car has four-wheel-drive, and the road barrier is substituted with a wall. Would it be possible to rise the front of the car by driving slowly against the wall?



7. Mass-spectrometer(9 p)

In the figure below, a simplified scheme of a mass-spectrometer is given. It is a device for measuring the masses of molecules. The substance under investigation is ionised by heating up to a temperature T on a hot filament (molecules undergo a single-electron ionisation). The ions are accelerated using voltage U . At first, let us neglect the thermal energy of the ions ($eU \gg kT$, where e is elementary charge and k — the Boltzmann's constant). A narrow beam of accelerated ions enters a region with magnetic field. For the sake of simplicity, let us assume that the region has a rectangular shape, and the magnetic field is homogeneous inside it. The magnetic field deflects the ions and depending on their mass, they may hit the detector. Let us assume that those ions which hit the centre of the detector enter and exit the region with magnetic field perpendicularly with its boundary, and the distance

between the entry and exit points is l (see figure)



1) Express the mass M of those ions which hit the centre of the detector via the quantities B , l , U and e .

2) The entrance to the detector is a circle with radius r ($r \ll l$). Because of the finite size of the detector entrance, ions within a mass range from $M - \Delta M$ to $M + \Delta M$ will enter the detector; find the range width ΔM .

3) Under the assumptions of the previous question, what is the range of the exit angles $\Delta\varphi$ of those ions which can still hit the detector?

4) Now let us assume that r is negligibly small, but we cannot neglect the thermal effects (still $eU \gg kT$). Due to difference in energies, the detector can be hit by ions of different masses, within the range from $M - \delta M$ to $M + \delta M$. What is the resolving power δM of the mass-spectrometer?

8. Optics experiment (10 p)

Equipment: a cylindrical bottle filled with water and having a millimetre scale across half of its perimeter (seen when looking through the bottle); measuring tape.

1) Determine, how long arc of the bottle's measuring scale could be seen simultaneously if looking at it through the bottle from a very distant point (much larger than the bottle's diameter), assuming that the observation point lies at the height of the millimetre scale.

2) Using the results of the previous experiment determine the angular radius of a rainbow (the angle between a ray coming to the observer's eye from a rainbow, and the axis of the cone formed by other such rays).

Remark: Rainbow is formed due to those rays which enter a spherical water droplet, reflect once from its surface internally, and exit

the droplet after second refraction. Note that the internal reflection is only partial, and not total, see figure. The exit angle α has a maximum as a function of the impact parameter b ; the angular radius of the rainbow equals to the maximal exit angle [indeed, if light of intensity I_0 falls onto the droplet with all possible impact parameters $b < r$, the light energy per impact parameter range Δb is $2I_0\pi b\Delta b$; hence, the energy per exit angle interval $\Delta\alpha$ is $\Delta I/\Delta\alpha = 2I_0\pi b\Delta b/\Delta\alpha = 2I_0\pi b(d\alpha/db)^{-1}$, which diverges near the maximum of the function $\alpha(b)$.]

