Estonian-Finnish Olympiad - 2011

1. Spool *(12 points)* A spool with inner radius r and outer radius R lies on a horizontal table; the axis of the spool is horizontal. A weightless rope is wound around the inner part as shown in the picture. The loose end of the rope makes an angle α with the horizont (the angle α can be also negative). The moment of inertia of the spool is J and mass — M. In what follows you may assume that the spool rolls on the table without slipping.

i) (2 pts) We pull the loose end of the rope with velocity u (parallel to the loose part of the rope; that loose part can be thought to be very long). What is the velocity of the spool?

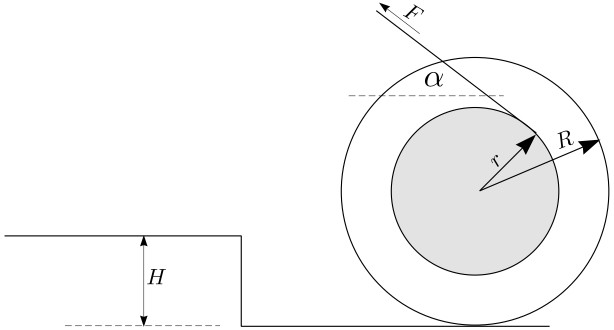
ii) (3 pts) Suppose now that the spool is at rest, and we apply force F to the loose end of the rope (parallel to the loose part of the rope). What is the acceleration of axis of the spool?

iii) (2 pts) How large does the coefficient of friction μ need to be (as a function of α) to ensure that there is no slipping between the spool and the table?

iv) (1.5 pts) Now the spool rolls, again, with velocity u; however there is no rope. The spool hits a threshold of height H (see Figure); the impact is perfectly inelastic. What is the speed v of the spool immediately after the impact?

v) (1.5 pts) What is the speed w of the spool after rolling over the threshold? Assume that u is such that the spool will roll over the threshold without losing contact with its edge.

vi) (2 pts) If the speed u is too large, $u > u_0$, the spool will jump up and lose contact with the edge of the threshold. Determine u_0.



2. Capacitor *(6 points)* An ideal plate capacitor has plates with area A and separation d and is charged so that the electric field between the plates equals to E.

i) (2 pts) Find the energy density of the electric field inside the capacitor and the total energy of the field.

ii) (1.5 pts) What is the force required to keep the plates separated?

iii) (2.5 pts) Now, this capacitor is submerged into distilled water of dielectric permittivity $\varepsilon = 80$; the electric field between the plates equals still to E. What is the hydrostatic pressure between the plates if the atmospheric pressure is p_0 and the pressure of the water column can be neglected?

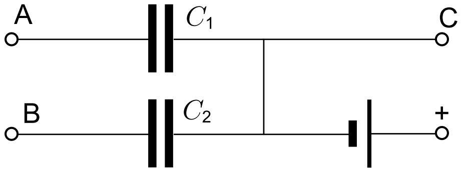
3. Charged cylinder *(8 points)* Dielectric cylinder of radius r carries a charge of surface density σ on its cylindrical surface and rotates with angular velocity ω.

i) (3 pts) Determine the magnetic induction B inside the cylinder. *Remark:* if you wish, you can use the expression for the inductance $L = \mu_0 N^2 S/l$ of a solenoidal coil of radius r, length $l \gg r$, area of cross-section S and number of loops N.

ii) (3 pts) A radial conducting wire connects the axis of the cylinder with the cylindrical surface (it rotates together with the cylinder). Find the electromotive force (voltage) $\mathcal{E}$ between the ends of the wire.

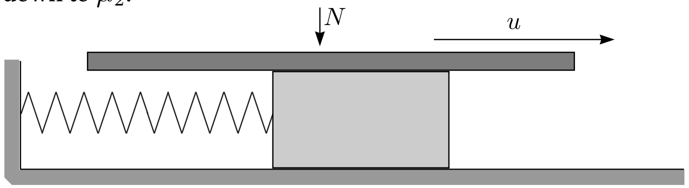
iii) (2 pts) Suppose that the wire connecting the axis of the cylinder with the cylindrical surface is not radial and has an arbitary shape (still, there are no segments protruding outside the cylinder). Show that $\mathcal{E}$ does not depend on the shape of the wire.

4. Black box *(10 points)* Equipment: a black box with three terminals, voltmeter, timer. Inside the black box, there are two capacitors and a battery, connected as shown in Figure. The capacitance $C_1 = (3400 \pm 400)\mu$F; you are asked to determine the capacitance C_2 and estimate the uctertainty. *Remark:* the terminal "+" is a wire, long enough to be conected to either terminal "A" or terminal "B".



5. Plutonium decay *(3 points)* Plutonium is an unstable element, a Pu^{239} atom decays with a half-life of $\tau_{1/2} = 24\,000$ years by creating smaller nuclei, including an α-particle. Find the α-particle flux density (i.e the number of passing nuclei per unit time and per unit cross-sectional area) near the surface of a plate of Pu^{239}. The plate has thickness $d = 1\,\mathrm{mm}$; its width and length are much larger than that. The density of plutonium $\rho = 19\,800\,\mathrm{kg/m^3}$. *Remark:* half-life is the period of time it takes for a substance undergoing decay to decrease in size (in the number of particles) by half. The mass of an atom of Pu^{239} is $m_0 = 3.84 \times 10^{-25}\,\mathrm{kg}$.

6. Violin string *(9 points)* The motion of a bow puts a violin string into a periodic motion. Let us make a simplified model of this process. The string has elasticity and inertia, so we substitute it by a block of mass m, fixed via a spring of stiffness k to a motionless wall and laying on a frictionless horizontal surface. The bow is substituted by a horizontal plate, which is pressed with constant force N downwards, and which moves with a constant velocity u, parallel to the axis of the spring, see Figure. The static coefficient of friction between the plate and the block is μ_1, and the kinetic coefficient of friction is $\mu_2 < \mu_1$. So, as long as the plate does not slide with respect to the block, the coefficient of friction equals to μ_1; as soon as there is some slip, it decreases down to μ_2.



i) (2 pts) For questions (i) and (ii), let us assume that the speed of the plate u is very small as compared to the maximal velocity of the block. What is the maximal velocity of the block $v_{\max}$ (maximized over time)?

ii) (2 pts) Sketch qualitatively the graph of the displacement of the block as a function of time and indicate on the graph the durations of the prominent stages of the block motion (graph segments).

iii) (1,5 pts) Now, let us abandon the assumption about the smallness of u. Sketch qualitatively the graph of the velocity of the block as a function of time.

iv) (2,5 pts) Determine the amplitude A of the block's oscillations.

v) (1 pt) Which condition (strong inequality, $\gg$ or $\ll$) must be satisfied for u in order to ensure that the oscillations will be almost harmonic?

7. Vacuum bulb *(8 points)* Let us study how a vacuum can be created inside a bulb by pumping. Let the volume of the bulb be V, and the pump consist in a piston moving inside a cylinder of volume αV, where $\alpha \ll 1$. The pumping cycles starts with piston being pulled up; when the pressure inside the cylinder becomes smaller than inside the bulb, a valve V_A (connecting the cylinder and the bulb) opens and remains open as long as the piston moves up. When piston is released, it starts moving down, at that moment, the valve V_A closes. As long as the valve V_A is open, the pressures of the bulb and the cylinder can be considered as equal to each other. When the piston moves down, the pressure in the cylinder increases adiabatically until becoming equal to the outside pressure $p_0 = 10^5\,\mathrm{Pa}$; at that moment, another valve V_B opens leting the gas out of the cylinder. When the piston reaches the bottommost position, there is no residual air left inside the cylinder. Now, the piston is ready for being lift up: the valve V_B closes and V_A opens, marking the beginning of the next pumping cycle. The air inside the bulb can be considered isothermal, with the temperature being equal to the surrondin temperature T_0. The adiabatic exponent of air $\gamma = c_p/c_V = 1.4$.

i) (2 pts) How many pumping cycles N needs to be done to reduce the pressure inside bulb from $p = p_0$ down to $p = \beta p_0$, where $\beta \ll 1$?

ii) (2 pts) What is the net mechanical work done during such a pumping (covering all the N cycles)?

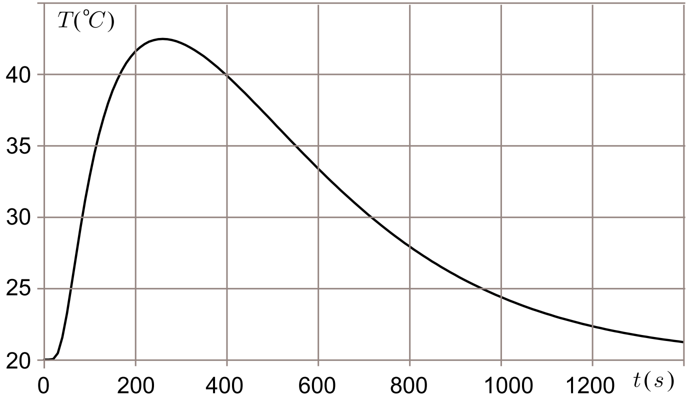
iii) (2 pts) What is the temperature of the air released from the cylinder to the surroundings at the end of the pumping process (when the pressure inside the bulb has become equal to βp_0)?

iv) (2 pts) According to the above described pumping scheme, there is a considerable loss of mechanical work during the period when the piston is released and moves down. Such a loss can be avoided if there is another pump, which moves in an opposite phase: the force due to outside air pressure pushing the piston down can be transmitted to the other pump for lifting the piston up. What is the net mechanical work done when such a pumping scheme is used?

8. Heat sink *(6 points)* Consider a heat sink in the form of a copper plate of a constant thickness (much smaller than the diameter d of the plate). An electronic component is fixed to the plate, and a temperature sensor is fixed to the plate at some distance from that compnent. You may assume that the heat flux (i.e. power per unit area) from the plate to the surrounding air is proportional to the difference of the plate temperature at the given point (the coefficient of proportionality is constant over the entire plate, including the site of the electronic component).

i) (2 pts) The electronic component has been dissipating energy with a constant power of $P = 35\,\mathrm{W}$ for a long time, and the average plate temperature has stabilized at the value $T_0 = 49\,^\circ\mathrm{C}$. Now, the component is switched off, and the average plate temperature starts dropping; it takes $\tau = 10\,\mathrm{s}$ to reach the value $T_1 = 48\,^\circ\mathrm{C}$. Determine the heat capacity C (units $\mathrm{J/^\circ C}$) of the plate. The capacities of the electronic component and the temperature sensor are negligible.

ii) (4 pts) Now, the electronic component has been switched off for a long time; at the moment $t = 0$, a certain amount of heat Q is dissipated at it during a very short time. In the Figure and Table, the temperature is given as a function of time, as recorded by the sensor. Determine the dissipated heat amount Q.



t (s)	0	20	30	100	200	300
T (°C)	20.0	20.0	20.4	32.9	41.6	42.2
t (s)	400	600	800	1000	1200	1400
T (°C)	39.9	33.4	27.9	24.4	22.3	21.2

9. Coefficient of refraction *(10 points)* *Equipment:* A thick glass plate having the shape of a half-cylinder, a glass prism, a container with an unknown liquid, a laser pointer, graph paper, ruler.

i) (5 pts) Determine the coefficient of refraction of the half-cylindrical glass plate and estimate the uncertainty of the result.

ii) (5 pts) Determine the coefficient of refraction of the liquid and estimate the uncertainty of the result.