Seagull–December 2015

1. (*BADMINTON*) What is the horizontal flight distance of a shuttlecock if it is launched horizontally at the edge of a cliff, from the height of $h = 800\,\text{m}$ with initial speed $v_0 = 50\,\text{m/s}$. The mass of the shuttlecock is $m = 5\,\text{g}$, the friction force between the shuttlecock and air is proportional to the shuttlecock's speed,$F_f = Cv^2$, with $C = 0{,}002\,\text{kg/m}$.

2. (*PENDULUM*) A body of mass $m = 0{,}1\,\text{kg}$ lies on a horizontal smooth surface and is connected to the wall via a weightless spring of stiffness $k = 50\,\frac{\text{N}}{\text{m}}$. Friction coefficient between the body and the surface $\mu = 0.01$. The body is displaced parallel to the spring so that the spring is deformed by $x_0 = 10\,\text{cm}$, and then released. How long time t will it take for the body to stop permanently? Free fall acceleration $g = 9{,}8\,\frac{\text{m}}{\text{s}^2}$.

3. (*VENTILATION*) A human produces by breathing approximately $m = 1\,\text{kg}$ of carbon dioxide during $T = 24\,\text{h}$. Let such a human work alone in an office room of volume $V = 30\,\text{m}^3$. Initially, the concentration of the carbon dioxide in the room was equal to that of the outside atmosphere, 400 molecules of CO_2 per 10^6 air molecules. How long will it take for the concentration of carbon dioxide to reach the value of 1000 particles per 10^6 air molecules? Due to ventilation, the exchange rate of the interior and exterior air is $\dot{w} = 10\,\text{m}^3/\text{h}$. Assume that the air coming from the ventilation system is mixed very fast with the air inside the room. The density of carbon dioxide $\rho = 1{,}9\,\text{kg/m}^3$.

4. (*MOON*) What would be the temperature in the middle of a celestial body of the size of our Moon in Kelvins if the following assumptions can be made. The celestial body consists of an homogeneous solid material of average density $\rho = 3\,\frac{\text{g}}{\text{cm}^3}$ and heat conductivity $k = 3\,\text{W/m K}$, and has the shape of a sphere of radius $R = 2000\,\text{km}$. It radiates heat as a perfectly black body, i.e. the heat flux density at its surface $w = \sigma T^4$, where $\sigma = 5{,}67 \cdot 10^{-8}\,\text{J/s m}^2\,\text{K}^4$ and T is its surface temperature. There is no external heat flux falling onto the surface of this celestial body. Due to nuclear decay of various isotopes (mostly uranium-238 and its decay products), the power density released in the crust material $P = 7 \cdot 10^{-12}\,\text{W/kg}$.

5. (*SPHERICAL MIRROR*) A concave perfectly reflecting mirror has the shape of a hemisphere of radius $r = 10\,\text{cm}$. A wide parallel beam of light falls onto such a mirror (the mirror is entirely inside the beam). The reflected light is collected by a circular photosensor of surface area $S = 1\,\text{mm}^2$. The total power of the collected light energy depends on the position of the sensor. What is the maximal percentage of the light energy which can be collected in such a way, from the total light energy which falls onto the entire mirror?

6. (*LATTICE*)

A square lattice is made from a wire; the small lattice squares have side length $d = 1\,\text{cm}$, and the resistance of the wire connecting two neighbouring nodes $R = 1\,\Omega$. The entire lattice consists of 101×101 small squares. What is the total charge which flows through a fictitious line connecting the centre of the lattice with its edge (the dashed line in the figure), if an external magnetic field (perpendicular to the plane of the lattice) of strength $B = 1\,\text{T}$ is switched on?