Problem Induction Motor
Deadline April, 1 2013

Induction (or asynchronous) motors are the simplest and most reliable electric motors. They are powered by alternating current, and they do not contain commutators, slip rings or brushes. They consist of a *stator* and a *rotor* (see fig. 1). The stator is a fixed set of coils, which produces a rotating magnetic field in the plane perpendicular to the axis of the motor. The rotor is just a cage, i.e., a set of closed metallic loops attached to the axis of the motor. The rotating magnetic field produced by the stator induces electric current in the loops of the cage, which behave as magnetic dipoles, and interact with the external field of the stator. As a result, a torque is exerted on the rotor, and it starts rotating.

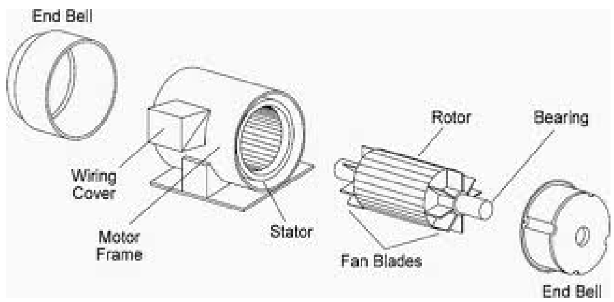

Figure 1: The structure of an induction motor

In a simplified model (see fig. 2) we assume that the magnetic induction vector $\mathbf{B}$ produced by the stator is rotating in the $x-y$ plane at a constant angular velocity $\mathbf{\Omega}$, and it has a constant magnitude $\mathbf{B}$. The axis of the rotor is in the z direction. The rotor is assumed to be a flat coil of area A, winding number N, Ohmic resistance R and self inductance L. The vector $\mathbf{n}$ perpendicular to this coil is rotating also in the $x-y$ plane.

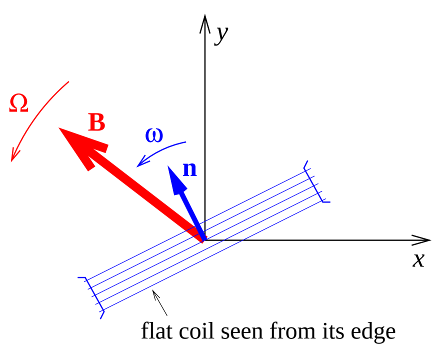

Figure 2: The simplified model, seen from the z axis

Stationary operation

First we study the stationary operation of the motor, when its angular velocity ω and torque T are constant. As we shall see, when the motor is loaded, its angular velocity ω gets smaller than Ω. It is convenient to characterize this shift by the *slip*

$$s = \frac{\Omega - \omega}{\Omega},$$

which is a dimensionless number between 0 and 1.

Stationary operation at small load

a. Assume that the load is small, so the slip $s \ll 1$. Determine the time average of the torque T exerted by the motor as a function of the slip s. (Use reasonable neglections.)

Stationary operation at arbitrary load

b. Determine the average torque T as a function of an arbitrary slip value s between 0 and 1.

c. Assume that $\frac{L\Omega}{R} = 10$. Sketch the graph of the function $T(s)$.

d. Find the maximal stationary torque $T_{\max}$ and the corresponding slip s_0.

Efficiency

e. Determine the efficiency η of the motor as a function of the slip s. (Assume that there is no energy loss due to friction, radiation.)

Stability

f. Assume that the motor is connected to an equipment (load) which has a linear $T_l(\omega)$ characteristics, so for a stationary angular velocity ω a torque $T_l(\omega) = K\omega$ is needed, where K is a positive constant. Determine graphically the stable and unstable operation points on the $T(s)$ graph obtained in question c) for different K -s.

Negative slip

g. Do the negative slip values have any physical meaning? If yes, what? If no, why?