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4: Atmospheric evaporation by Jeans escape

Thermal atmospheric escape is a process in which small gas molecules reach speeds high enough to escape the gravitational field of the Earth and reach outer space. Particles in a gas collide with each other due to their thermal energy. These collisions constantly accelerate and decelerate particles, varying their speed continuously. Paticles might be accelerated until they reach the so-called escape velocity and leave the atmosphere. This process, known as Jeans escape, is critical for the formation and maintenance or the evaporation of the atmosphere of a planet. It is believed that it played an important role in the loss of water from Venus and Mars atmospheres, due to their lower escape velocity.

In this problem we will quantify the scattering (collisions between gas particles) and rate of loss of hydrogen in Earths atmosphere. The modulus and direction velocity distribution of the molecules of a gas of mass m and at a temperature T is given by the Maxwellian distribution:

$$f(v)d^3v = (\frac{m}{2\pi kT})^{\frac{3}{2}} \exp(-\frac{mv^2}{2kT})\ v^2\ \sin(\theta)\ dvd\theta d\varphi \tag{1}$$

where d^3v is the velocity differential. In spherical coordinates it is expressed as $d^3v = v^2\ \sin(\theta)\ dvd\theta d\varphi$.

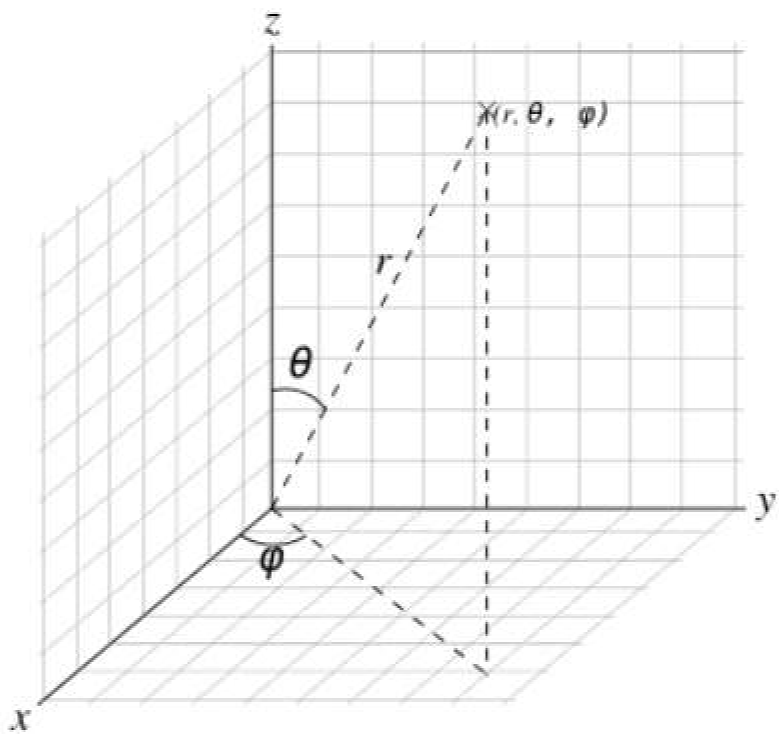

Figure 1: Schematic illustration of the spherical coordinate

Thus at any temperature there can always be some molecules whose velocity is greater than the escape velocity. A molecule located in the lower part of the atmosphere would not, in general, be able to escape to outer space even though its velocity is greater than the limit velocity because it would soon collide with other molecules, losing a big part of its energy. In order to escape, these molecules need to be at a certain height such that density is so low that their probability of colliding is negligible. The region in the atmosphere where this condition is satisfied is called exosphere and its lower boundary, which separates the dense zone from the exosphere, is called exobase. The temperature at the exobase is roughly 1000 K.

1 Exobase height

Exobase is defined as the height above which a radially outward moving particle will suffer less than one backscattering collision on average. This means that the mean free path has to be equal to the scale height, which is defined as the height where the atmospheres density is $\frac{1}{e}$ lower than on Earths surface ($R_E = 6.37 \times 10^6 m$). The mean free path λ is the average distance covered by a moving particle in a gas (that we consider to be ideal) between two consecutive collisions and this can be expressed by the following equality:

$$\lambda(h) = \frac{1}{\sigma n_V(h)} , \qquad (2)$$

where σ is the effective cross sectional area for the collision hydrogen atom-atmosphere $\sigma = 2 \times 10^{-19} m^2$ and n_V is the number of molecules per unit volume. Atmospheres density decreases with exponentially from altitude 250 km:

$$P(h) = P_{Ref} exp(-\frac{(h - h_{Ref})}{H}) , \qquad (3)$$

where we know that at an altitude of 250 km, the pressure is 21 μ Pa. H is the scale height, and its value is $H = 60\ km$.

(a) Determine the air particles mean free path λ at the altitude of 250 km. (0.8 points)

(b) Determine the exobase height h_{EB}. (1.2 points)

2 Atmospheric escape flux

Particles in the exobase with enough outwards velocity will escape gravitational attraction.

(a) Assuming a Maxwellian distribution, determine the probability that a hydrogen atom has a velocity greater than the escape velocity in the exobase. (2.0 points)

(b) Determine the hydrogen atoms flux (number of particle per unit area and per unit time) Φ that will escape the atmosphere, knowing that the concentration of hydrogen atoms in the exobase is $n_H = 10^{11} m^{-3}$. Be careful with the dimensions (2.0 points)

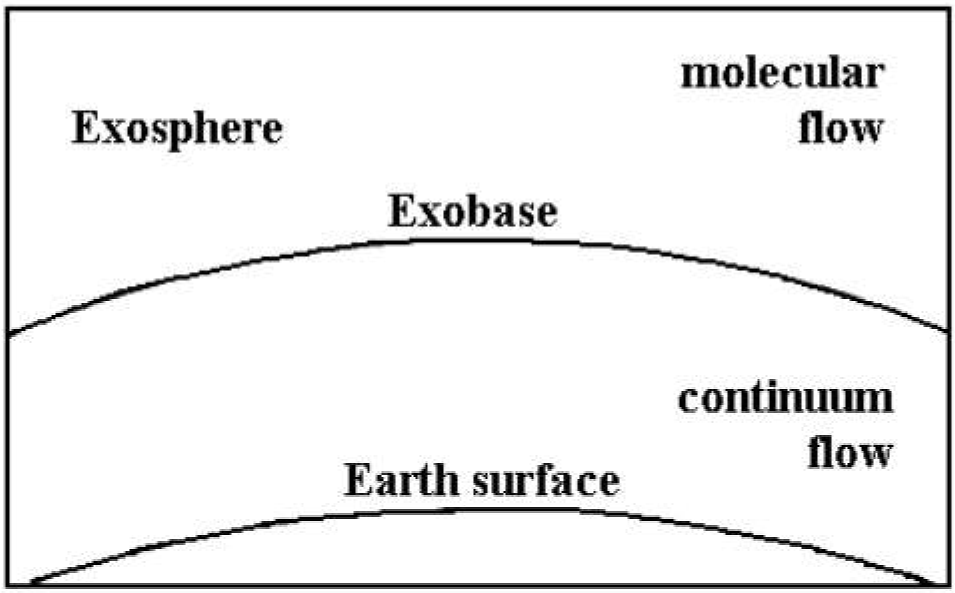

Figure 2: Diagram showing different zones of the atmosphere. In exosphere, particles with high enough velocities may leave the atmosphere

3 Evaporation of the atmosphere

Thermal atmospheric escape is one of the processes that explain why some gases are present in the atmosphere and some others are not. Currently the atmospheric pressure is approximately $P_0 = 10^5 Pa$ and a fraction of $\chi_H = 5.5 \times 10^{-5}\%$ of the atmosphere molecules are hydrogen molecules. When these molecules reach a certain height (lower than h_{EB}) they split into two atoms due to solar radiation. Concentration of hydrogen atoms in the exobase can be considered constant over time.

(a) Knowing that the average molar mass of the atmosphere is $M_{Atm} = 29\frac{gr}{mol}$, estimate the number N_H of hydrogen atoms present in the Earths atmosphere. Assume that the gravity near earth surface is $9.5\ \frac{m}{s^2}$ (1 points)

(b) Find out how much time would it take for half of the hydrogen atom to escape the Earths atmosphere. (1.5 points)

(c) Find also this time for Helium atoms on Earth, knowing that their concentration in the exobase is $n_{He} = 2.5 \times 10^{12} m^{-3}$. Helium atoms are the $5 \times 10^{-4}\%$ of the atmosphere. (1.5 points)

Jeans escape is not the only atmospheric escape process and there are also other reactions on the surface on Earth that may produce atmospheric gases. Yet this calculations should show the difference in orders of magnitude of the escape of different gases