Helical Rope

A device is built by placing two metal strips, each of mass m, on a large, frictionless cylinder. Two identical massless elastic ropes each with spring constant k and obeying Hooke's Law are used to connect the metal strips such that the two ropes are initially at their natural length x_0 and parallel to each other. The contact points of each rope on the same strip are diametrically opposite to each other, and the whole device can be seen on Figure 1. Strip A is bolted to the cylinder, while strip B is free to move along and rotate about the cylinder's axis.

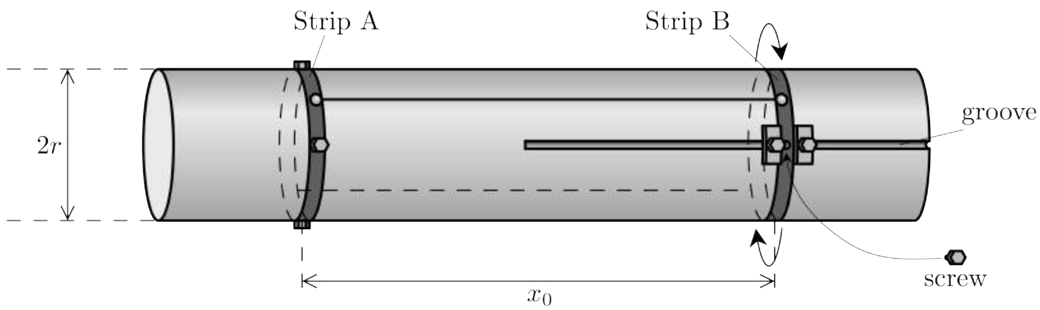

Figure 1: The device in its initial configuration. The screw can be used to prevent rotation of strip B.

1. The cylinder is now oriented such that its axis is vertical under a constant gravitational field g, and strip A is above strip B. Strip B is now rotated N times while its distance from strip A is maintained at x_0. After this, strip B is prevented from rotating by a screw, as shown in Figure 1.
   (a) Find an equation that, given numerical values for the initial conditions, would allow you to calculate x_1, the new equilibrium position.
   (b) Under certain conditions the metal strip will undergo simple harmonic oscillation. Calculate the frequency of oscillation for small Δx, in terms of k, r, N, x_0, and x_1.
2. The cylinder is now oriented horizontally, the ropes are returned to their initial position, and strip B is prevented from rotating by means of the screw.
   (a) A horizontal stretching force F is now applied to strip B. If the force is increased very gradually, the ropes break when the force reaches a value F_0. What, then, is the minimum amount of constant force required to break the ropes?
   (b) If strip B is rotated N times before the screw is put in place keeping the total length of the ropes x_0, calculate the minimum horizontal force required to break the ropes if said force is:
      i. increased very gradually.
      ii. kept constant.

3. The system is returned to its initial horizontal configuration. All constraints on strip B are now removed, and the strip is rotated through an angle θ_0 while keeping the distance between the two metal strip x_0, and then released (initially $\dot{x}(0) = 0$, and $\dot{\theta}(0) = 0$).
   (a) Find the equation of motion of strip B!
   (b) Solve the equation of motion for $x\,(t)$ and $\theta\,(t)$!
   (c) Find the maximum velocity and maximum angular velocity, and also the time T required for strip B to reach strip A!