Motion in the electrostatic field of a Dipole

The characterization of point object motion, when both radial and tangential forces are applied, is usually rather complicated, and requires advanced mathematical tools. However, some systems such as a motion of a point charge near an electric dipole, which have a very specific electrostatic field, provide interesting results, even for the case when an angular momentum is not conserved.

Assume that the relativistic and the electromagnetic radiation effects can be neglected, unless otherwise stated.

1 Movement of the dipole

Start with a case, where a point small charged object with a charge $+Q$ is fastened to the table. The center of the dipole is fixed at the distance L from the charged object (see Figure 1). The dipole consists of two identical small balls fastened to the tiny, rigid rod with a length d, $d \ll L$, so that the moment of inertia can be ignored. Each of the balls has a mass m and have charge $+q$ and $-q$. The dipole can rotate around its center in a plane parallel to the surface of the smooth table.

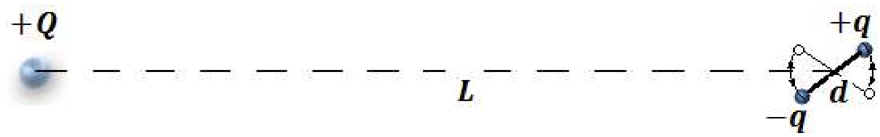

Figure 1: Schematic representation of the system used in section 1.1

1. Calculate the period of the small oscillations T of the dipole around its stable equilibrium axis in the electrostatic field of the charged object.

   Now, the dipole freely moves around the charged object, which is still fastened to the table. The dipole is launched with an initial velocity $\mathbf{u}$, as shown in Figure 2. The parameters of the system are chosen in such a way that the period of oscillation of the dipole in the electrostatic field of the fixed object is large enough to assume that the dipole is always oriented along the line between the dipole and the charged object.

2. Determine the magnitude of tangential v_{t1} and normal v_{n1} components of the velocity of the dipole in terms of Q, q, m, d, u, L and r . (r is the distance from the center of the dipole to the charged object).

   In order for the dipole to get closer to the charged object, its initial velocity should be less than some critical value $u < v_{cr}$.

3. Find this critical initial velocity v_{cr}.

4. Sketch the trajectory of the center mass of the dipole for the case when the dipole is launched with the critical initial velocity v_{cr}, considering a very long time (radiation effects take place).

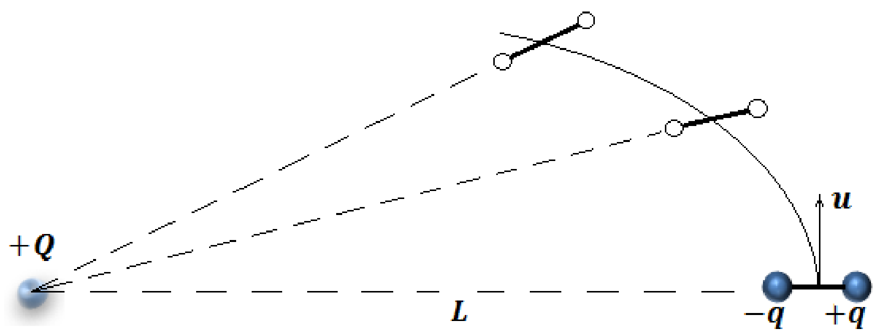

Figure 2: Top view of the case, when the dipole is moving around the fastened charged body. (Not to scale)

Suppose that the condition $u < v_{cr}$ is applied and radiation effects are very small.

5. What time t_1 is required to reduce the distance between the dipole and the charged body to half of the original distance?

2 Motion around the fixed dipole

In this part, analyze a situation when an angular momentum is not conserved. The system is the same as in the previous part with the only difference that the dipole is fixed and the charged small object with a mass $2m$ is moving around the dipole. The electrostatic field of the dipole is easier to describe in the polar system of coordinates, which is defined with the distance r from the center of the dipole, and angle θ counted counterclockwise, as shown in Figure 3.

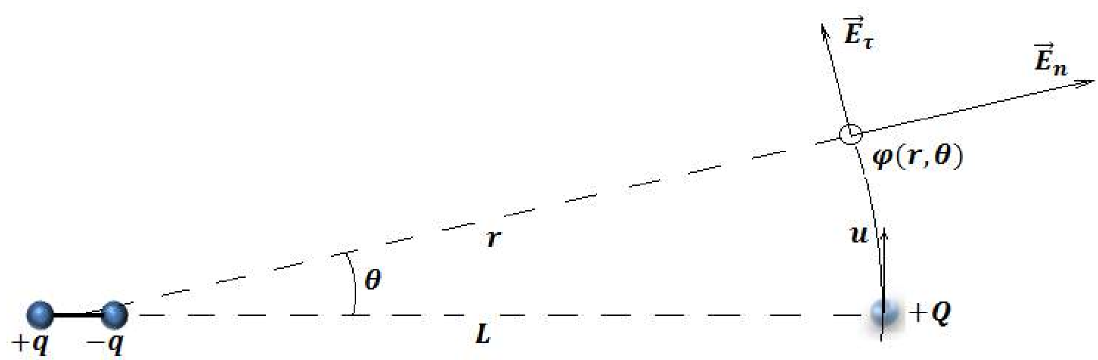

Figure 3: The system analyzed in Part 2. (Direction of the vector $\mathbf{E_n}$ and $\mathbf{E_t}$ could be wrong)

1. Determine the electrostatic potential φ at a distance $r \gg d$ from the dipole, as a function of θ.
2. Find the components of the electric field E_n and E_τ in terms of r, θ, q and d.

3. What torque is applied to the moving object counting from the center of the dipole when the object is at the distance r and angle θ from the dipole?
4. Determine tangential component of the velocity $v_{\tau 2}$ of the charged object as a function of coordinates r and θ. Hint: $\frac{d\theta}{dt} = \omega$ angular velocity
5. Calculate the normal component of the velocity v_{n2} of the moving object.

   Now it is time to compare results with the first part.
6. Find the time t_2 to reduce the distance between the dipole and the charged body to $L/2$.

3 Circular motion

In this part the peculiarity of the circular motion around the dipole is analyzed. Initially, the system is the same as in Part 2, with the exception that the charged object is connected to the center of the dipole with a light, rigid insulating rod with the length L. This rod easily rotates around the axis, which is perpendicular to the surface of the table. Thus, the charged object moves around the dipole along circular trajectory with radius L.

1. What is the maximum and minimum velocity of the charged object v_{max} and v_{min} during circular motion around the dipole?
2. Derive an expression for the force N acting from the moving object on the rod in terms of m, L, Q, q, d and u.
3. With what initial velocity u_c should the charged object be launched, so that it will move around the dipole along a circular trajectory, even without the rigid rod?
4. Sketch the orbit of the charged object for the situation described in 3 after a long time (radiation effects make their impact on the motion).

   Useful math : $\int \frac{x dx}{\sqrt{A - x^2}} = -\sqrt{A - x^2} + C$, where A and C are some constants