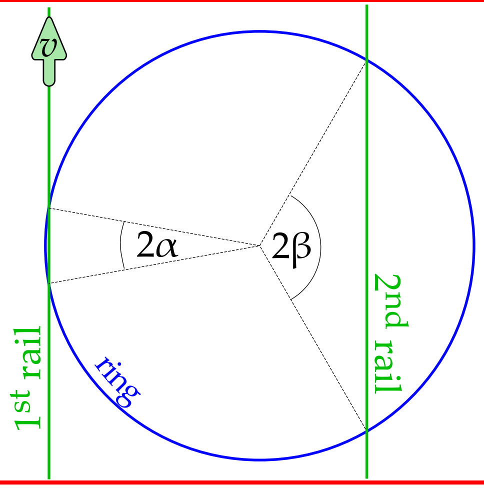
Problem 7

A homogeneous ring lays horizontally on two identical parallel rails. The first rail moves parallel to itself, with a constant speed v; the second rail is at rest. The angular distance between the ring-rail contact points, as seen from the centre of the ring, is 2α for the first rail, and 2β for the second rail, see figure. Assuming that $\alpha \ll 1$ and $\beta = \pi/3$, find the speed of the centre of the ring.



Hints after 1st week: This problem can be solved by using a brute force approach, i.e. writing down two equations for two unknown angles. However, the solution can be significantly simplified once a useful geometrical fact is noticed: then, it is enough to write down only one equation for one unknown quantity.

Hints after 2nd week: Typically in the case of static's problems, it is convenient to start with a torque balance, because the origin for the balance equation can be chosen in such a way that arms of at least two forces become equal to zero; also, you are free to choose, which forces you want to disappear from your torque balance (which are the least desirable). In particular, if there are only three forces applied to a rigid body at an equilibrium, the lines along which these forces are applied intersect always in a single point. Here, in order to derive the "geometrical fact" mentioned in the previous hint, study the torque balance with respect to the intersection point of the lines along which two forces (e.g. the friction forces due to the 1st rail) are applied.

Hints after 3rd week: Here is the "to-do-list" for the Problem 7. Let the x-axis be along the rails, y-axis — the other horizontal axis, and z — the vertical one. Do not put $\alpha = 0$; instead neglect terms which are much smaller than α^2 (e.g. using $\cos\alpha \approx 1 - \alpha^2/2$). Find the magnitudes of the friction forces using the torque balance in y-z-plane. Study the torque balance of two friction forces in x-y-plane (e.g. those due to the 1st rail) with respect to the intersection point P of the lines defined by the remaining two friction forces. As a result, you should be able to notice that the position of the point P defines the directions of all the four friction forces. Write the x-directional force balance equation of the friction forces using the distance L of the point P from the 1st rail as a single unknown quantity; solve the equation using the approximation $\alpha \ll 1$ (be careful: L has the same order of smallness as α^2!). Once you know L, the ring's speed can be easily found by finding first the distances between the instantaneous rotations centres of the ring in the reference frames of the 1st and of the 2nd rail, and the centre of the ring. Finally note that numerically approximate answer can be found using purely geometrical constructions, e.g. with the help of GeoGebra applet (still, analytical exact result is much more preferred).

Results of the 7th problem.

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Correct solutions have been submitted by