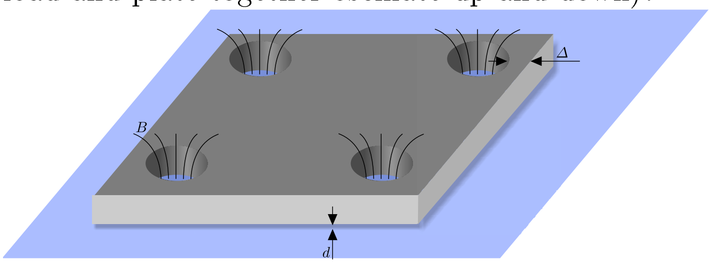
Problem 10: magnetic levitation

A rectangular superconducting plate of mass m has four identical circular holes, one near each corner, see figure. Each hole carries a certain magnetic flux (all the four fluxes are equal and of the same polarity). The plate is put on a horizontal surface which is also in a superconducting state. The magnetic push between the plate and the surface compensates the weight of the plate when the width of the air gap beneath the plate is d, which is much smaller than the distance between the plate's and holes' edges (denoted by Δ in figure); d is also much smaller than the radii of the holes.

When the plate levitates in such a way above the support, the frequency of its small vertical oscillations is ν_0. Next, a load of mass M is put on the plate, so that the load lays on the plate, and the plate levitates above the support. What is the new frequency ν of small vertical oscillations (when the load and plate together oscillate up and down)?



Hints after the first week. Due to the IPhO in Copenhagen, there are no hints this week (many contestants are attending IPhO and will not have internet access this week). Publication of the solutions of Problem 9 is delayed due to the same reason by one week.

Hints after two and half weeks. Study the solution of the Problem 1, Part C, question ii from the 43rd IPhO.

Hints after the third week. Find the repulsive force as a function of distance d: first calculate the magnetic field energy as a function of d, and further apply the virtual displacement method.

Hints after the fourth week. Let us notice that between the plates, B-field is horizontal, i.e. perpendicular to the vertical axis z (why?), and independent of the z-coordinate (why?). The value of the $B(x,y)$-field can be found using the Gauss law. In the neighbourhood of a circular hole, this is an easy task, because the contribution of the remote holes can be neglected (hence, the problem becomes cylindrically symmetric). The problem becomes more complicated in the region where the distance to all the holes has the same order of magnitude. Luckily, there is no need for us to solve that problem exactly, because the shape of the field-lines between the plates is independent of the distance between the plates (as long as $d \ll \Delta$); try to motivate this! Now, suppose that for a certain value of $d = d_0$, there is a certain dependence $B_0(x,y)$ (which we don't know). Then, using the Gauss law, we can express $B(x,y)$ for any value of d ($d \ll \Delta$) in terms of $B_0(x,y)$, d_0, and d. Hence, we can also express the magnetic field energy in terms of $B_0(x,y)$, d_0, and d, which makes it possible to find the force $F = F(d)$ using the virtual displacement method.