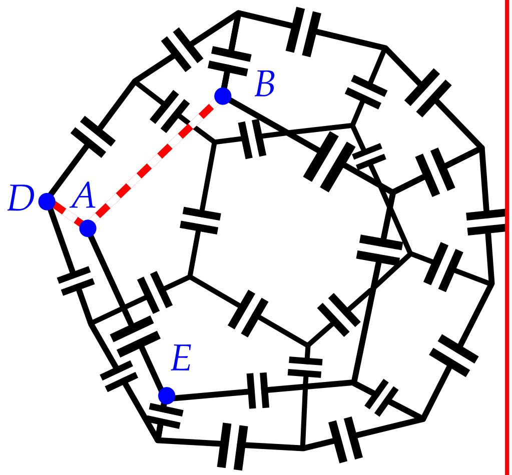
Problem 6

Edges of a dodecahedron are made of wire of negligible electrical resistance; each wire includes a capacitor of capacitance C, see figure. Let us mark a vertex A and its three neighbours B, D and E. The wire segments AB and AD are removed. What is the capacitance between the vertices B and E?



Hints after 1st week: This problem has also **a short solution which does not use brute force**. How to be sure that you have found the short solution: using the method of that short solution, it is possible to solve also a modified problem, where the dodecahedron is replaced by an infinite honeycomb lattice (two wires are cut off in the same way as for this dodecahedron).

Hints after the 2nd week: As a first step, find the resistance between B and E when the segments DA and AB (together with the respective capacitors) are still present. This can be found in the same way as the resistance r between two neighbouring nodes P and Q of an infinite square lattice of resistors R: consider the superposition of two current distributions. (i) current I is driven into the node P and driven out symmetrically at infinity; (ii) current is driven into the lattice at infinity, and out from the node Q. Due to symmetry, in both cases there is a current $I/4$ in the wire directly connecting P and Q. For the superposition, current I enters the circuit at P, and leaves from Q, and there is a current $I/4 + I/4 = I/2$ in the wire connecting P and Q, i.e. $r = R \cdot (I/2) / I = R/2$.

Hints after the 3rd week: In the case of a dodecahedron, current I, if driven into a node P, cannot be driven out at infinity because the circuit is finite. However, there is still a way to drive it out from the nodes of the dodecahedron so that (i) the current distribution remains symmetric; (ii) for a superposition of two such current distributions of opposite polarities (when adding a current distribution with I being driven out from a node Q), the external currents driven to and from all the nodes other than P and Q cancel out.

Now, suppose you know the resistance r between the nodes B and E for a uncut dodecahedron: between B and E, the whole uncut dodecahedron is equivalent to a single resistor r. Next, notice that cutting out a resistor R is mathematically equivalent to adding a parallel resistance $-R$ (a mathematician doesn't care that there are no negative resistances).

Intermediate conclusion after the 3rd week.

Correct solutions have been submitted by (ordered according to the arrival time):

1. Lars Dehlwes (Germany)*

2. Hrishikesh Menon (India)

3. Ly Nguyen (Vietnam) **(short solution was submitted later, order nubmber 14a)**

4. Dan-Cristian Andronic (Romania)

5. Szabo Attila (Hungary)

6. Jan Ondras (Slovakia)

7. Ng Fei Chong (Malaysia)

8. Tudor Ciobanu (Romania)** **(short solution was submitted later, order number 13a, i.e. between Kongas and Schmidt)**

9. Vũ Việt Linh (Vietnam) **(short solution was submitted later, order number 14b)**

10. Madhivanan Elango (United Kingdom)

~~11. Nguyen Ho Nam (Vietnam) (short solution was submitted later, order nubmber 14c)~~ **Second week begins here**

12. Jaemo Lim (Korea)***

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14. David Schmidt (Germany)***

15. Efim Mazhnik (Russia)***

16. Colibaba Nicoleta (Moldova)**

*** Solution includes a typo at the very last line** ****Correct version submitted at the second attempt**

***** Short solutions**