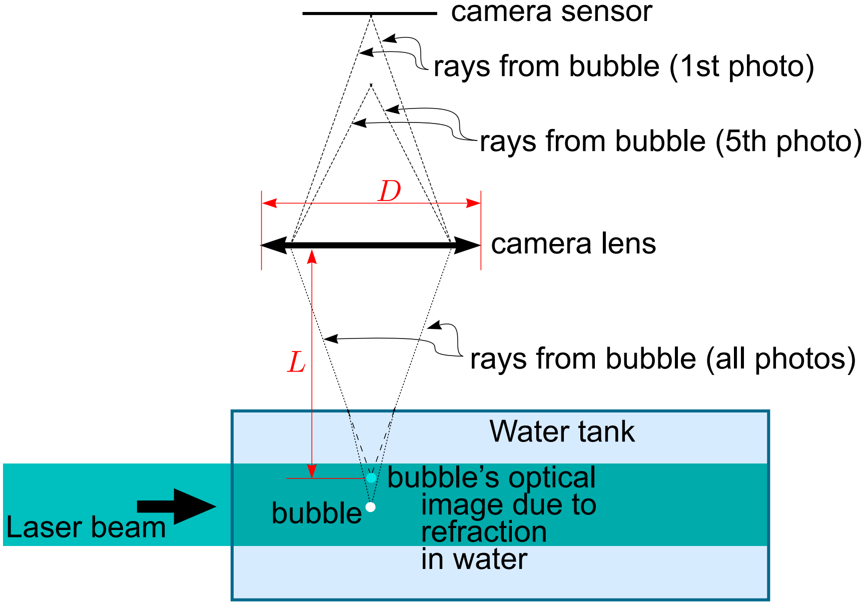
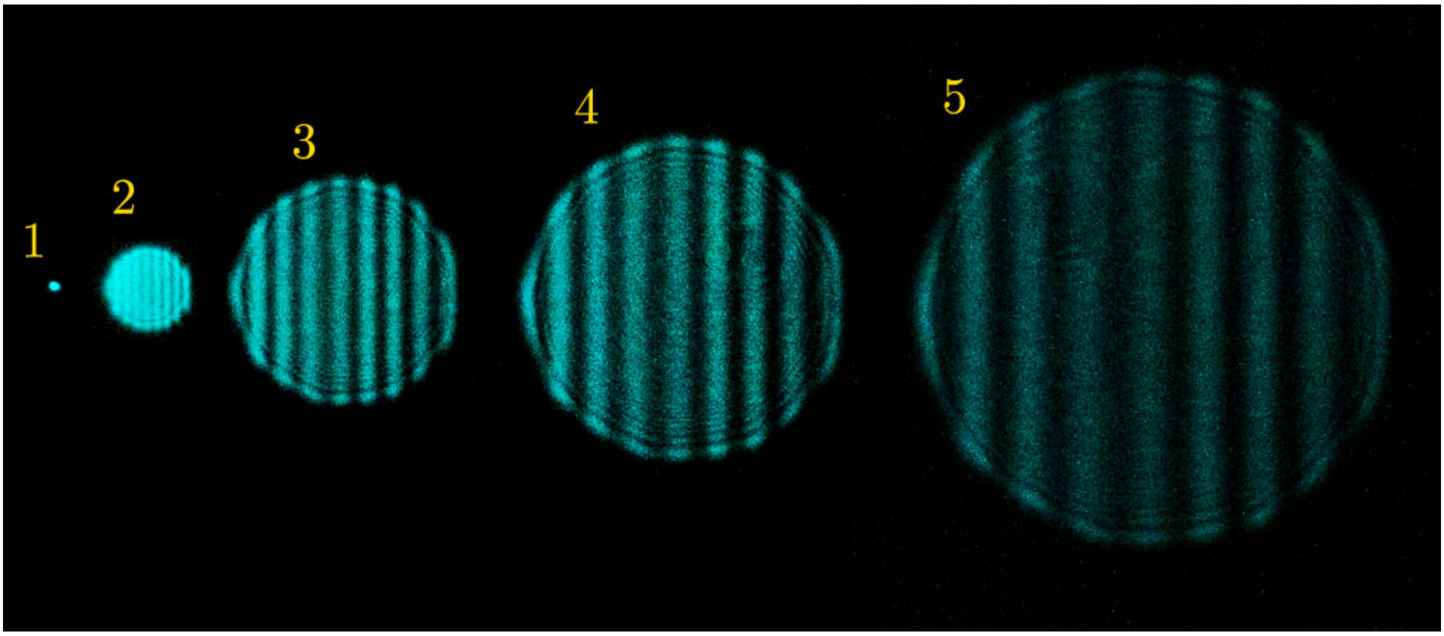
Problem 8: gas bubble in water (contributed by Mihkel Kree)

Introduction. People living in colder climates have surely noticed that by filling a glass with cold tap water one gets a glass of misty (or rather milky) water. The reason is that depressurizing and warming of the water causes the initially dissolved gas to come out of the solution and form tiny bubbles. In this problem you are going to calculate the size of such gas bubble in water.

A photographer prepared a setup consisting of a rectangular water tank with glass walls, a laser beam entering the water tank perpendicularly to one of its faces, and a camera looking directly towards a neighbouring face of the water tank. A gas bubbled entered the laser beam and the photographer managed to take five photos of the bubble while continuously defocusing the camera. The lens had "internal focusing" design, so that defocusing meant changing the focal length while keeping the position of the lens intact, see figure. The line of sight from the camera to the bubble was perpendicular to the laser beam, and the bubble was entirely inside the beam.



In the figure below, the taken photos are placed side by side and indicated by numbers $1-5$.
Task: calculate the diameter of the gas bubble.
Parameters: index of refraction of water with respect to gas: $n = 1.3$; wavelength of the laser: $\lambda = 488$ nm; the lens of the camera can be considered as a single convex lens with focal length $f = 10$ cm and diameter $D = 3.6$ cm (the change of the focal length due to defocusing was less than 10%); the distance from the bubble to the lens: $L = 30$ cm (more precisely, this is the distance from the lens to the image of the bubble as seen from the centre of the lens, see figure above).



Hints after the first week. If you have a glass ball, observe, what you can see when it is illuminated by a point source (a lamp) from a side. Alternatively, you can study the photo at
http://en.wikipedia.org/wiki/File:Clayton_Anderson_zero_g.jpg: (from where the light comes from?).
Hints after the second week. We can observe here a nice system of regularly periodic diffraction stripes. Such a diffraction pattern can be observed for two-slit diffraction, but there are clearly no slits in the case of this experimental setup. However, a similar pattern can be observed if there is an interference of light rays coming from two coherent point sources, assuming that the size of the screen on which we observe the interference pattern is much smaller than its distance to the point sources. *(Indeed, if there*

is a cylindrical screen, and the two point sources are at the axis of the cylinder, the diffraction pattern on the screen will be exactly the same as on a flat screen behind two-slits; if we straighten a small piece of a cylindrical screen the change of its shape is small, and hence, the change of the diffraction pattern on it is also small.) So, we can make an hypothesis that the diffraction pattern is due to two point sources which are created by the light scattering effects of the bubble. Since the diffraction pattern is very clean (minima are very dark), one can conclude that almost all laser light reaching the lens comes exclusively from those two point sources, and that they have nearly equal brightness.

Hints after the third week. There are three apparent candidates for the point sources responsible for the interference pattern. These are images of the laser created (a) via a reflection from the convex bubble surface (which works as a convex mirror, except that the surface is only partially reflecting); (b) via a reflection from the concave bubble surface (in which case the laser light refracts into the bubble, is reflected by the concave surface, and refracts back to the water); (c) via two sequential refractions from the air-water interface of the bubble. Closer inspection shows that via two sequential refractions, the light cannot be diverted as much as by 90 degrees (this would require a larger value of the refraction index). Please pay attention that the bubble cannot be considered as an ideal lens: you can find (and make use of) an image for a narrow beam of light hitting the bubble with an impact parameter a, but the position of the image (and the effective focal length) is a function of a. Finally, please note that for this problem it may happen that you obtain an equation which needs to be solved numerically.

Correct results have been submitted by:

1. **Oliver Edtmair** (Austria), 3.168 pts
2. **Kuo Pei-Cheng** (Taiwan), 2.880 pts
3. **David Stein** (Germany), 2.356 pts
4. Dan-Cristian Andronic (Romania), 1.403 pts
5. **Szabó Attila** (Hungary), 1.731 pts
6. **Jakub Safin** (Slovakia), 1.967 pts
7. Lars Dehlwes (Germany, 1.464 pts
8. Andres Põldaru (Estonia), 1.331 pts
9. Vũ Viêt Linh (Vietnam), 0.871 pts
10. Ly Nguyen (Vietnam), 0.713 pts
11. Cyuan-Han Chang (Taiwan), 0.72 pts
12. Madhivanan Elango (United Kingdom), 0.792 pts
13. Kurenkov Mikhail (Russia), 0.9 pts
14. Ismael Salvador Mendoza Serrano (Mexico), 0.513 pts.

(Bold font marks the recipients of the best solution award, $e^{-1/5}$ each.)

Solutions

The solutions can be divided into two classes: (a) based on the calculations of optical path lengths, and (b) based on the calculation of the distance between two images (effective point sources) created by the reflections from the bubble' surface.

There were three solutions submitted before any hints, and all these were quite well written. Oliver Edtmair submitted his (a)-type solution already by Monday night.

Kuo Pei-Cheng's (a)-solution (submitted Wednesday) could have benefited from few more explanations, but his drawings and calculations were very clear and self-explanatory. David Stein's (b)-type solution (submitted Sunday night) was the first one to show that it is impossible for a ray to reach the camera purely by refractions, without reflections from the bubble's surface (on behalf of those who didn't: if there were three rays reaching the camera, the diffraction pattern would have been non-periodic). However, he did mistake in counting the number of stripes (as a rule, any mistake will result in a non-acceptance of the solution, but such really minor mistakes are exceptions; though, they incur a penalty factor of 0.9).

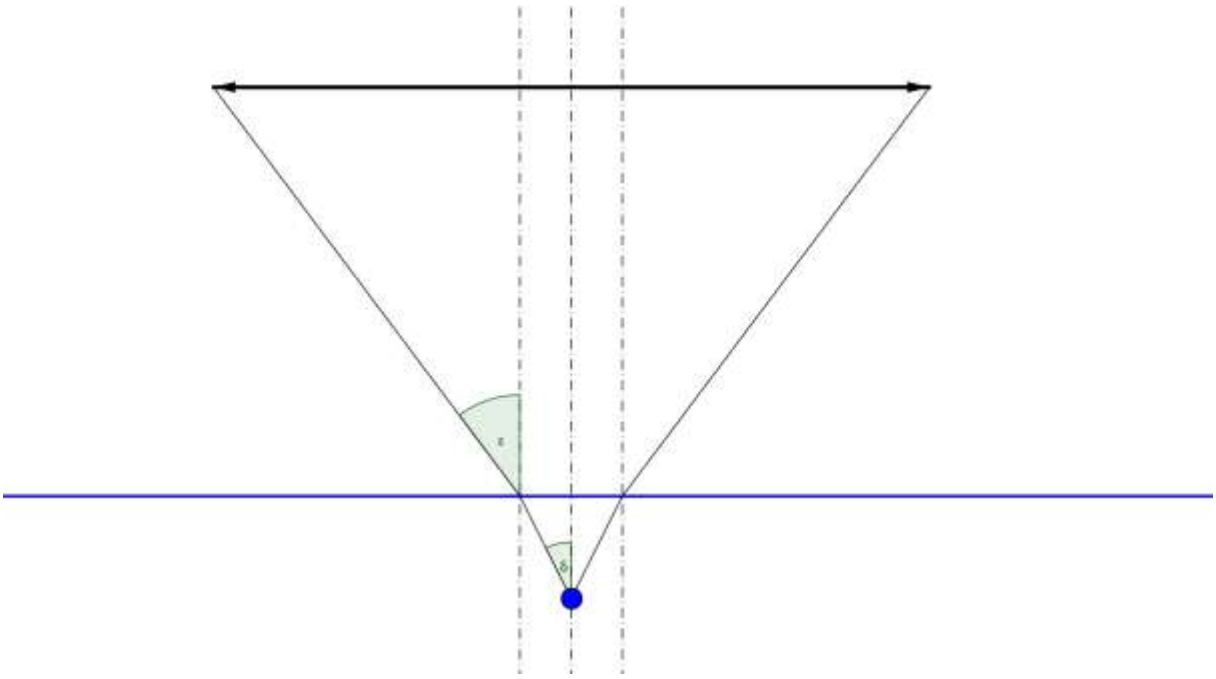
Apart from these first-week-solutions, the best-solution-awards went to Attila Szabó, who was the first one to make an error analysis, and to Jakub Safin, who documented clearly all the approximations and assumptions which were made during the calculations.

Below, all these solutions are attached in the submission order. This covers all the main solving techniques, so that there is no need for providing additional solutions.

Final words

The Problems 7, and 8 were really difficult, and the Problem 9 is difficult, too. I promise, Problem 10 will be a simpler one. So, stay online on 7th July, otherwise you may miss the opportunity of getting a speed bonus! — JK

Problem 8



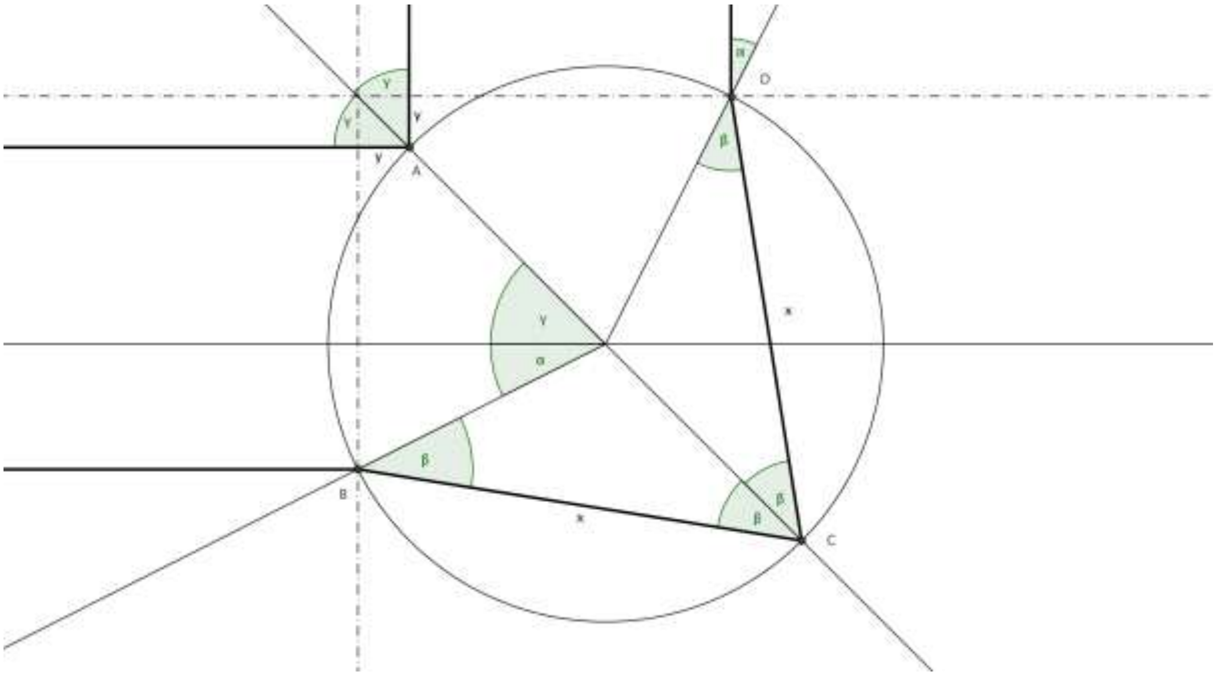
Light rays within a small angle δ to the line perpendicular to the water surface pass through the lens.

$$\frac{\sin\delta}{\sin\varepsilon} = \frac{1}{n}$$

$$\tan\varepsilon = \frac{D}{2L}$$

$$\delta \approx 0.046\, rad$$

The light rays are refracted and reflected by the water-gas surface. There are two possible ways for the light rays in order to leave the bubble within the angle δ.



One ray is reflected at point A, the other one enters the bubble at point B, is reflected at point C and leaves the bubble at point C. For each γ an angle α can be found so that the two rays leaving the bubble are parallel. These parallel rays meet in the picture and cause the interference pattern. The criteria for constructive interference is

$$k\lambda = 2x + \frac{\lambda}{2} - 2ny$$

$$k\epsilon\mathbb{Z}$$

Equations for x, y, α, β and γ (R … radius of the bubble)

$$x = 2R\cos\beta$$

$$y = R(\cos\alpha - \cos\gamma)$$

$$\gamma = 2\beta - \alpha$$

$$\frac{\sin\alpha}{\sin\beta} = \frac{1}{n}$$

The equations of constructive interference of two different bright lines on the picture can be subtracted.

$$\frac{\Delta k\lambda}{2} = \Delta x - n\Delta y$$

(Δk … number of dark lines between the two bright lines)

As δ is small, the changes in the angles α, β and γ are small either. Therefore

$$\Delta x \approx -2R\sin\beta_0\,\Delta\beta$$

$$\Delta y \approx R(-\sin\alpha_0\,\Delta\alpha + \sin\gamma_0\,\Delta\gamma)$$

(α_0, β_0 and γ_0 are the angles of the two rays leaving the bubble perpendicular to the water surface. These rays cause the bright line in the middle of the picture. The reference line is the outmost line. Thus $\Delta k = 4$)

$$\frac{\Delta k\lambda}{2} \approx -2R\sin\beta_0\,\Delta\beta - nR(-\sin\alpha_0\,\Delta\alpha + \sin\gamma_0\,\Delta\gamma)$$

$$\approx -2nR\sin\alpha_0\,\Delta\beta + nR\sin\alpha_0\,\Delta\alpha - nR\sin\gamma_0\,\Delta\gamma$$

$$\approx -nR\sin\alpha_0\,(2\Delta\beta - \alpha) - nR\sin\gamma_0\,\Delta\gamma$$

$$\approx -nR(\sin\alpha_0 + \sin\gamma_0)\Delta\gamma$$

$\Delta\gamma$ can be obtained from $\Delta\gamma = \frac{\delta}{2} \approx 0.023\,rad$

γ_0 is $\gamma_0 = \frac{\pi}{4}\,rad$

Equation for α_0

$$\gamma_0 = 2\sin^{-1}(n\sin\alpha_0) - \alpha_0$$

$$\alpha_0 \approx 0.47\,rad$$

The radius of the bubble is

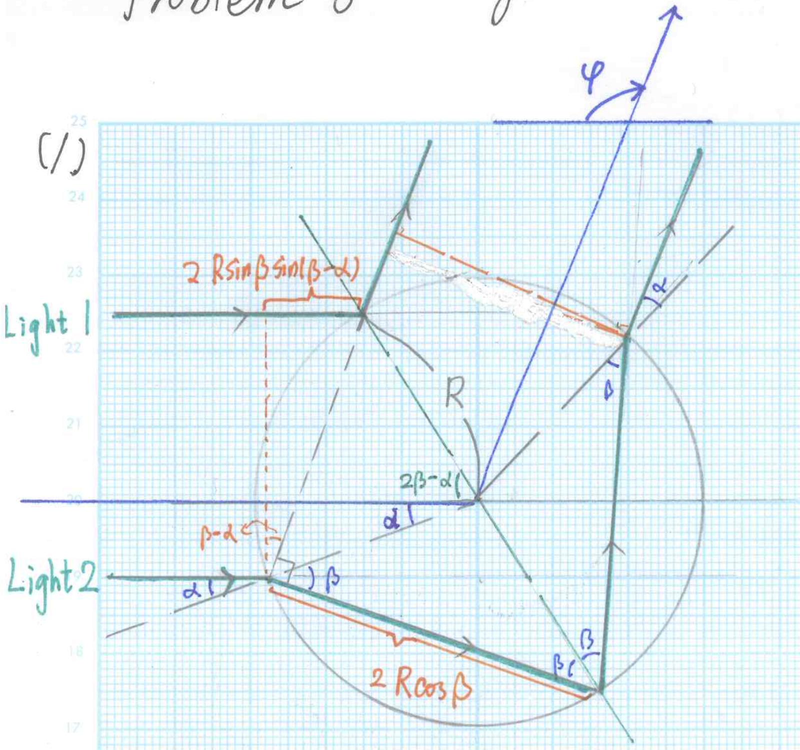
$$R \approx \frac{\Delta k\lambda}{2n(\sin\alpha_0 + \sin\gamma_0)\Delta\gamma} \approx 0.00003\,m$$

The diameter of the bubble is

$$d \approx 0.00006\,m \approx 60\,\mu m$$

Problem 8 gas bubble in water

(1)

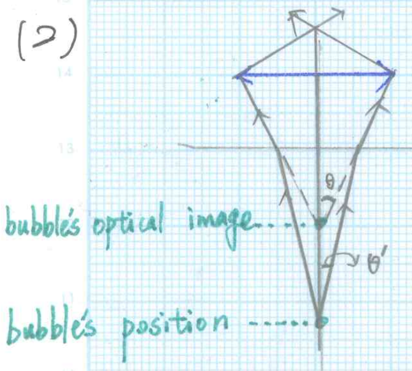


At φ direction, we could conside Light 1 and Light 2. Denote optical length difference between them by Δ

$$\Delta = 4R\cos\beta - 4R\sin\beta\sin(\beta-\alpha)\cdot n$$

$$= 4R(\cos\beta - n\sin\beta\sin(\beta-\alpha))$$

(2)



$$\tan\theta' = \frac{1.8}{30} \div 1.3 = 0.0462 \Rightarrow \theta' = 2.643°$$

$$\varphi = 4\beta - 2\alpha : \underset{(\varphi_1)}{\underline{87.36°}} \sim \underset{(\varphi_2)}{\underline{92.64°}}$$

(3)

α (°)	0	5	10	15	20	25	30	35	40	45	50	26.6°	26.11	27.52
β (°)	0	6.506	13.05	19.66	26.40	33.33	40.54	48.21	56.68	66.82	84.78	35.6°	34.90	36.92
$\varphi = 4\beta - 2\alpha$ (°)	0	16.02	32.19	48.65	65.60	83.30	102.2	122.9	146.7	177.3	239.1	89.2°	87.37	92.63
$\frac{\Delta}{4R}$	1	0.990	0.959	0.906	0.831	0.732	0.605	0.445	0.238	-0.05	-0.648	0.694	0.706	0.672

observe the taken photos, we can roughly find that $|\Delta(\varphi_1) - \Delta(\varphi_2)| = 8\lambda$

$$\Rightarrow \quad 4R(0.706 - 0.672) = 8 \times 488 \ (nm)$$

$$\Rightarrow \quad 2R = 5.74 \times 10^4 \ (nm) \approx \underline{0.06 \ (mm)}$$

Problem 8-Solution

We want to identify each ray by its number of interactions I with the gas bubble. Therefore a number of interactions I means 2 transmissions/refractions and I-2 reflections. In order to minimize loss of intensity we have to investigate rays with minimal number of interactions. We are looking for 2 rays, that can interfere with each other. Therefore one ray has to be the directly reflected with incident angle of 45°. Let α be incident angle and β be refraction angle. Each incident angle inside is β. That can be concluded by simple geometric analysis. By each reflection the ray rotates about $2\beta-\pi$, by each refraction/transmission the ray rotates about $\beta-\alpha$. Let the aperture be at an angle $\pi/2$ from line bubble-laser.

We have to minimize I, therefore we begin with I=2 (I=1 would mean one reflection):

I=2

Transmitted ray rotates about $2(\beta-\alpha)$ and that has to be $\frac{\pi}{2}$

(1) $\beta=\alpha+\frac{\pi}{4}$

(2) $n\sin\alpha=\sin\beta$

$$n\sin\alpha=\sin(\alpha+\frac{\pi}{4})=\sin\alpha\cdot\cos\frac{\pi}{4}+\cos\alpha\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}(\sin\alpha+\cos\alpha)$$

$$\Rightarrow\tan\alpha=\frac{1}{\sqrt{2}\cdot n-1}=1.193\Rightarrow\alpha=\arctan 1.193\approx 50^\circ\Rightarrow\beta=95^\circ$$

But this is impossible, because $\beta>90^\circ$ means the ray would be total reflected. Therefore I=2 is not of interest.

I=3

(1) $4\beta-2\alpha=\frac{\pi}{2}\Rightarrow\alpha=2\beta-\frac{\pi}{4}$ from simple geometric consideration

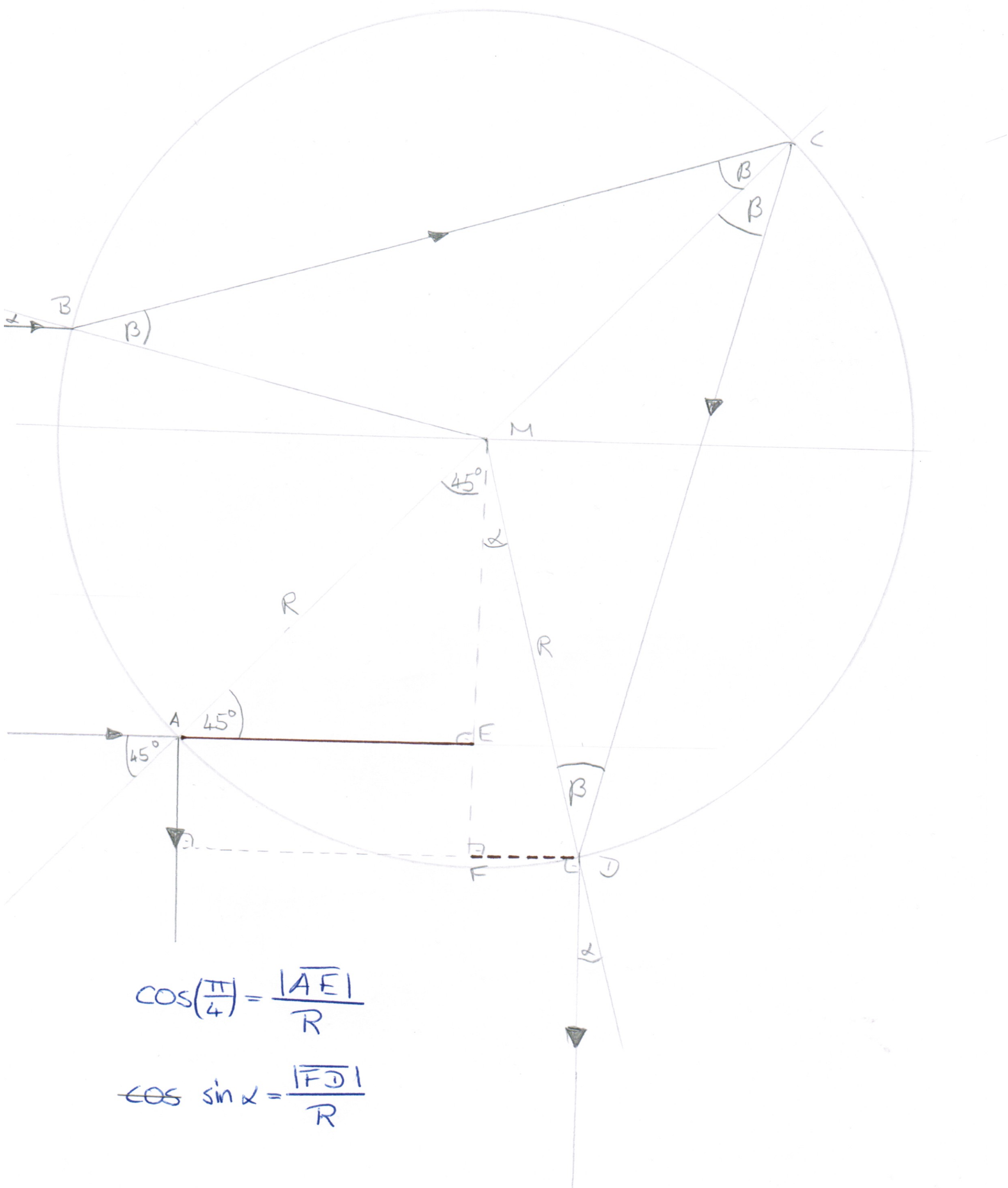
(2) $n\cdot\sin(2\beta-\frac{\pi}{4})=\sin\beta\Rightarrow f(\beta)=n\cdot\sin(2\beta-\frac{\pi}{4})-\sin\beta$

We solve this equation by using Newton's method:

$$\beta_{n+1}=\beta_n-\frac{f(\beta_n)}{f'(\beta_n)}$$

$$\beta_{n+1}=\beta_n-\frac{n\sin(2\beta_n-\frac{\pi}{4})-\sin\beta_n}{2n\cos(2\beta_n-\frac{\pi}{4})-\cos(\beta_n)}\quad,\quad\beta_0=40^\circ$$

We get the approximated angle $\beta=35.9^\circ$. It follows from this: $\alpha=26.8^\circ$. These angles seem to be the right ones because they satisfy the conditions stated above.



$$\cos\left(\frac{\pi}{4}\right) = \frac{|\overline{AE}|}{R}$$

$$\cancel{\cos}\ \sin\alpha = \frac{|\overline{FD}|}{R}$$

We need the distance of the first and the second ray. From geometry we can conclude, that:

$$d_r = |\overline{AE}| + |\overline{FD}| = R\left(\cos\frac{\pi}{4} + \cos\alpha\right) = \frac{d}{2}\left(\cos\frac{\pi}{4} + \sin\alpha\right)$$

$$d = 2\mathrm{d}_r \cdot \frac{1}{\cos\frac{\pi}{4} + \sin\alpha} \quad (3)$$

The interference pattern is established by two monochromatic point sources with distance d_r.
The interference pattern is very similar to the one produced by a lattice with lattice parameter d_r.

$$\Delta s = m \cdot \lambda = d_r \cdot \sin\gamma \quad ,$$

where γ is the angle under which a maximum is seen. Let δ be the aperture angle, under which light can be travel trough the lense. Obviously it is:

$$\tan\delta \approx \delta = \frac{D}{L} \quad ; \quad \delta \ll 1 \quad \text{because of} \quad L \gg D$$

If N is the total number of interference bands we can say, that unter the angle $\frac{\delta}{2}$ we can see the $\frac{N}{2}$ -th maximum. Therefore

$$\frac{N}{2} \cdot \lambda = d_r \sin\frac{\delta}{2} \approx d_r \cdot \frac{\delta}{2} \Rightarrow N\lambda = d_r \delta = \frac{d_r \cdot D}{L}$$

$$d_r = \frac{L \cdot N\lambda}{D} \quad (4)$$

Setting (4) into (3) we get:

$$d = \frac{2 \cdot L \cdot N \cdot \lambda}{D} \cdot \frac{1}{\cos\frac{\pi}{4} + \sin\alpha}$$

Now we want to set in the numerical values: $N = 9$, $\alpha = 26.8°$ and the given information from the text. Then we find:

$$d = \frac{2 \cdot 30\mathrm{cm} \cdot 9 \cdot 488\mathrm{nm}}{3.6\mathrm{cm}} \cdot \frac{1}{\frac{1}{\sqrt{2}} + \sin 26.8°} \approx 63.21\,\mu m$$

WoPhO Selection Round Problem 8
Physics Cup – Bubble in water
Attila Szabó, Grade 12
Leőwey Klára High School
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There are two phenomena happening to laser light which result in rays going perpendiularly to the incoming ones: reflection on the primary surface of the bubble; refraction into the bubble, reflection on its inner surface and refracting out of it. The two will have nearly equal intensities, hence the pattern will be sharp as in the photos.

In the first case, a perpendicular reflection will happen if the angle of incidence is $\alpha = 45°$. In this case, the reflected ray will intersect the optical axis in a distance of $AI = R\cos\alpha = R/\sqrt{2}$ from the centre of the bubble. Since the image will be formed by rays coming not in this plane but a slightly rotated one about the optical axis, this point should be considered as one of the point sources: since we're interested in rays coming in a small angle to this one, this apparent 'point source' is a good approximation in this plane (see Figure 1).

Now we're going to calculate the focal point of the bubble for the latter optical path. When refracted into the bubble, the angle of refraction will be $\beta = \arcsin(n \sin\alpha)$ by Snell's law. As OCD is an isosceles triangle, the angle of incidence and thus the angle of reflection at D is β as well, so is the angle of incidence at E due to the isosceles triangle ODE: at point E, the angle of refraction is α due to symmetry. Consider the quadrilateral $CDEM$ where M is the intersection point of the lines of the incoming and finally outgoing rays. By geometry, the angle at vertices C and E is $\beta - \alpha$, that at D is 2β, consequently, the angle at M which is apparently the angle of diversion is $360° - 4\beta + 2\alpha$; this angle must be either 90 or 270 degrees the two rays to be perpendicular. There is no solution for the first case in the trivial domain $0 \leq \alpha \leq 90°$; the second case gives rise to the solution $\alpha = 26.82°$ (calculated numerically); by checking the geometry we'll see that the ray should come from below the optical axis to leave upwards. Let the outgoing ray intersect the optical axis in K: since we're interested in rays going upwards this point can be considered as an effective focal point. It's easy to see that $AEK\angle = \alpha$ and $AKE\angle = 90°$; from the right triangle AEK one can see that K is $R\sin\alpha = 0.4512R$ away from the centre of the bubble and on the opposite side of A compared to I (see Figure 2).

Consequently, the laser will apparently be focused into two points when seen from the camera, the distance of which is $d = AK + AI = 1.1583R$, this will be the apparent distance seen by the camera as surface refraction changes only vertical distances. This means that the arising diffraction pattern will look like the pattern of two slits of distance d, thus the angular distance of neighboring minima are $\alpha = \lambda/d$.

To measure α on the photo, we need to know the angular diameter that is imaged in it. This is clearly $\beta = D/L$ as the light cone coming from the apparent point sources can get to the lens only in this angle. The apparent distances are proportional to angular distances in the slightly defocused images. The measured apparent diameter of the 5th spot (400% magnification in Adobe Reader) is $b = (235 \pm 3)$ mm, the apparent distance of 7th neighbor minima is $7a = (195 \pm 5)$ mm, thus the distance of neighboring minima is $a = (28 \pm 1)$ mm. The above mentioned proportionality yields that $a/b = \alpha/\beta$, by substituting everything:

$$\frac{a}{b} = \frac{\alpha}{\beta} = \frac{\lambda/d}{D/L} = 0.8634\frac{\lambda L}{RD} \to R = 0.8634\frac{\lambda L b}{Da} = (29 \pm 1.5)\ \mu\text{m},$$

thus the diameter of the bubble is $\delta = 2R = (59 \pm 3)\ \mu$m.

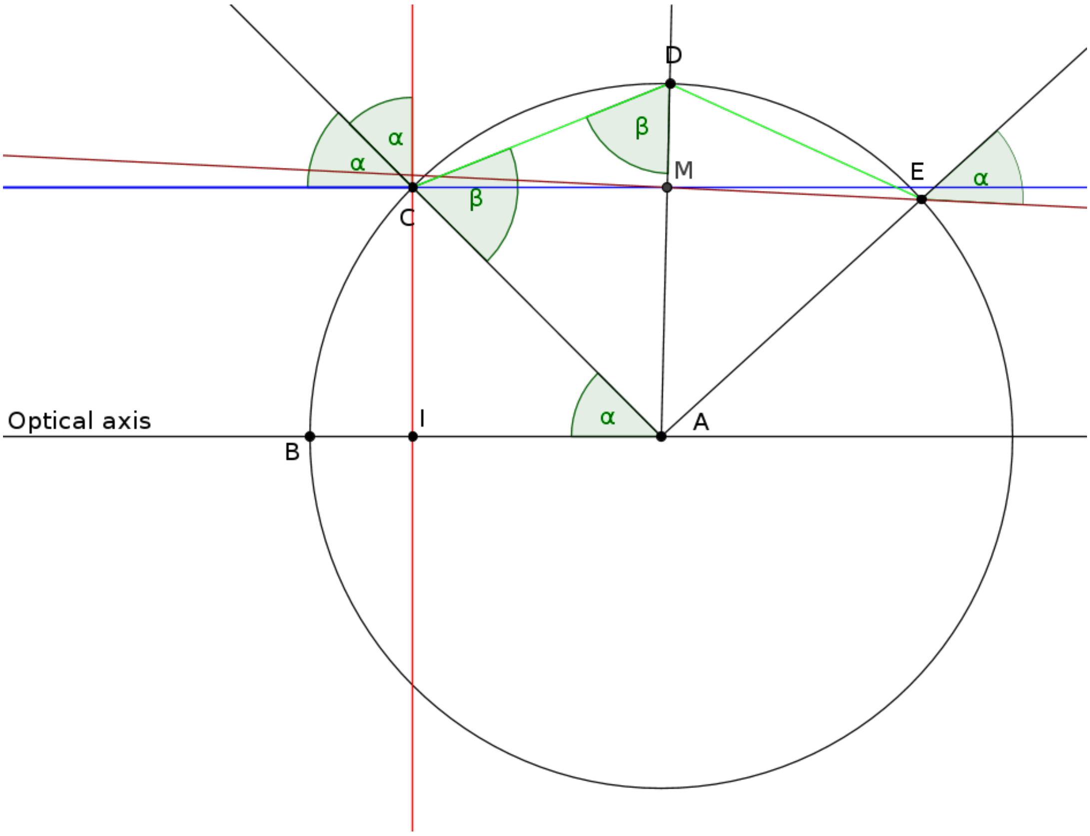


Fig. 1. The optical setup at $\alpha = 45°$.

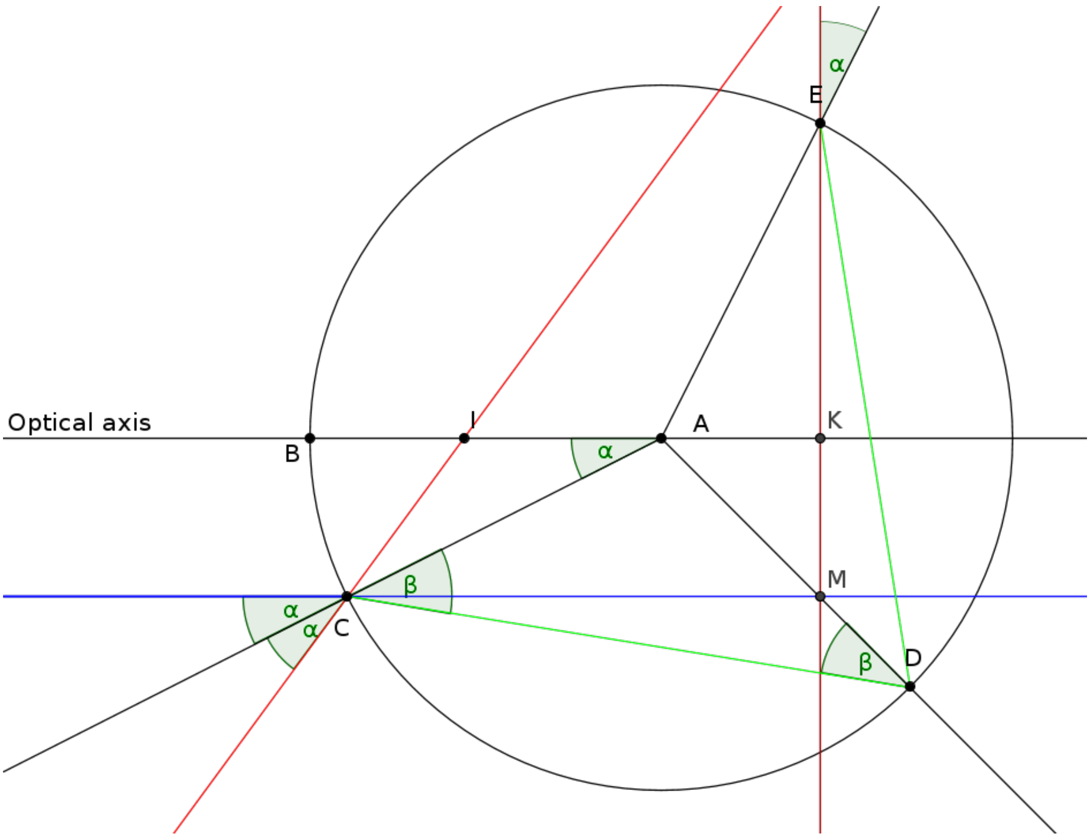
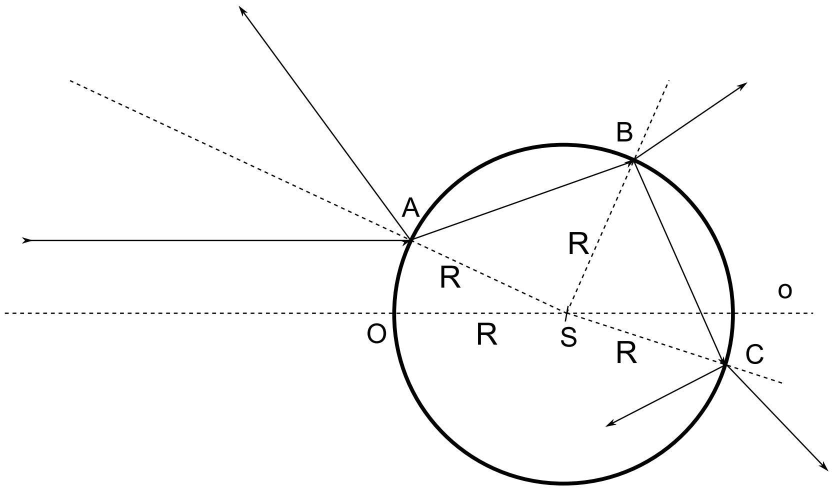


Fig. 2. The optical setup at $\alpha = -26.82°$.

In both figures, the blue ray is the incoming one, the green ones are the rays propagating inside the bubble, the red one is the reflected and the brown is the refracted-reflected-refracted back ray.

Jakub Šafin (Xellos)
WoPhO Physics Cup, problem 8

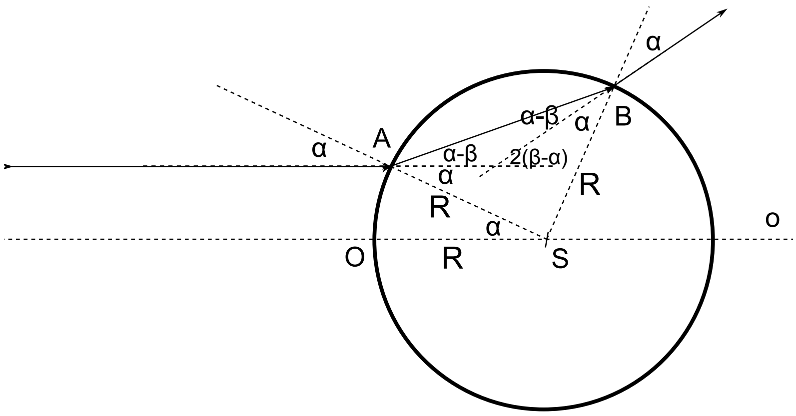
Consider a ray of the laser incident on the bubble. If the angle of incidence is large enough, total reflection may occur. Otherwise, the ray reflects partly from the outer bubble surface of the bubble, and it also refracts partly into the bubble. Then, the refracted part of the ray is once again incident on the inner bubble surface, and once again, it's partly reflected into the bubble and partly refracted out of it (full reflection never happens, because $n > 1$). The same process repeats itself with the reflected part of the ray - but since partly reflected part of the ray has much smaller intensity than the partly refracted one, we may neglect this effect. Then, a ray splits into 3 noticeable rays:



Now, let's utilize the fact that $D \ll L$: the photo comes only from parts of rays, which exit the surface of the bubble (either by first reflection or by any refraction) perpendicularly to their original rays, e.g. are diverted from the original path by an angle $\delta \approx 90°$ (the relative errors of the result coming from such an estimate are no more than several %).

First, let's check the double-refracted rays. We could see already that these aren't incident on the lens, because their intensity is very large (most of the original ray) and the interference pattern would be invisible. But let's calculate the angle δ for them.

Consider the center of the bubble on the optical axis o, and a ray $\parallel o$ incident on it at a distance $R \sin \alpha$ above o. More precisely now, we want $\delta \in [0, \pi]$ and we're not interested in the sign, because a ray incident at distance l below the axis has the same δ, just with opposite sign, so $\delta = 180° + \delta = -\delta$ for out purposes.

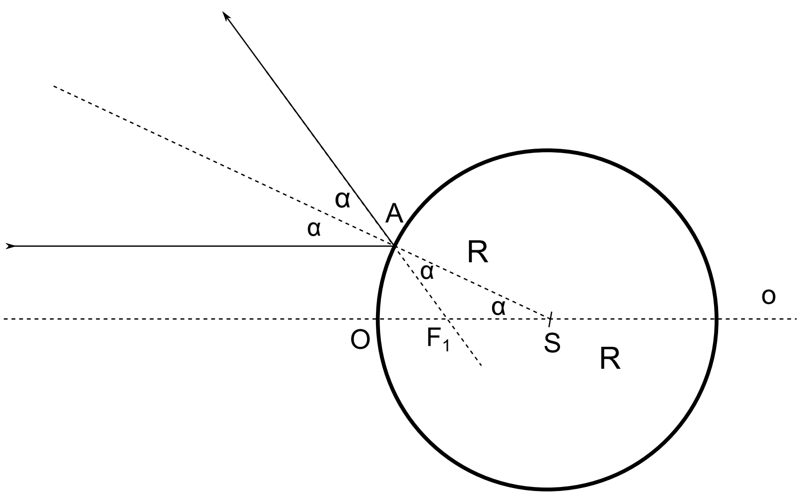


Utilizing isosceles triangles in a circle, we see that $\delta = 2(\beta - \alpha)$. By Snell's law, $\sin\beta = n\sin\alpha$. Then, the derivative of $\delta = 2(\arcsin{(n\sin{alpha})} - \alpha)$ by α is

$$2(\frac{n\cos\alpha}{\sqrt{1-n^2\sin^2\alpha}} - 1) \geq 2(n\cos\alpha - 1)$$

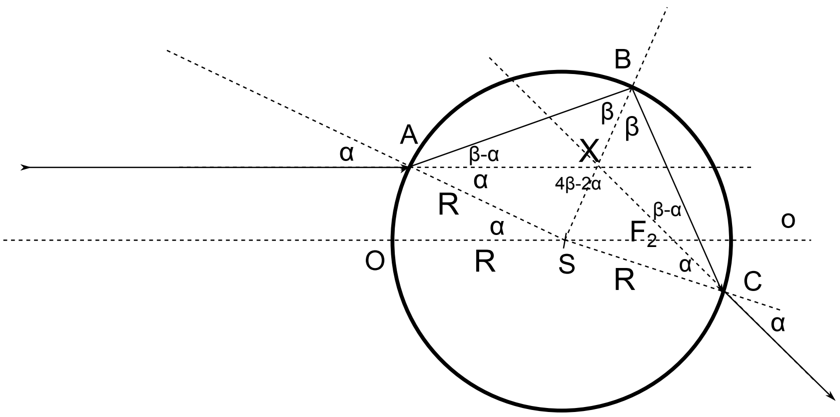
Now, let's jump to fully reflected rays? Total reflection happens in case the angle of incidence α satisfies $\sin\alpha > \frac{1}{n}$, so $\alpha > 50°$ in our case. So for the double-refracted rays, $\alpha < 50°$ and $\cos\alpha > 0.6$, so the derivative is always positive. This says that δ is maximum when α is maximum, e.g. the approx. 50 degrees, and then, we get $\delta = 80°$. So close! Well, those rays don't affect the picture, because their δ never even comes close to right angle.

Since we already have some info about fully reflected rays, let's give a trivial picture:



from which we see already that $\delta = 2\alpha$, which for $90° > \alpha > 50°$ gives $180° > \delta > 100°$. Those rays are out of the picture (in both meanings, lol) as well. And if we're looking for rays which just reflect partly and have $\delta \approx 90°$, then we want them to have $\alpha \approx 45°$, so they are centered at focus $F_1 \in o$, with distance $|OF_1|$ derivable by extending the reflected ray to reach o; those rays are perpendicular to the original ones and therefore to o, so $\triangle ASF_1$ is isosceles and right, and then $|OF_1| = R - \frac{R}{\sqrt{2}} = 0.29R$.

This point is the first source of the interference pattern. There are just the rays which refract, reflect and refract again, left. For those, we draw the image



and see that $\delta = |\angle AXC| = 4\beta - 2\alpha > 0$ ($90° > \beta > \alpha > 0$ for $n > 1$, from monotonicity of sine). We've seen already that for increasing α, $2(\beta - \alpha)$ is increasing as well, and so is β (monotonicity of sine again). There is no nice analytical solution to the equation $4\beta - 2\alpha = 90°$, but when we know it's increasing, we can produce a Python script to binary-search the right α:

```
from math import *

def expr(a): # a: alpha in radians; 4b-2a > PI/2?
return 4*asin(1.3*sin(a))-2*a-PI/2

PI =3.142; soclose =10**-5
aa =0; ab =49*PI/180 # minimum alpha, maximum alpha
# watch out for ab > 50*PI/180

while ab-aa > soclose:
ac =(aa+ab)/2
if expr(ac) > 0: ab =ac
else: aa =ac

print(aa)*180/PI # alpha in degrees
```

which gives $\alpha = 27°$. Those rays are centered at focus F_2. Then, we may derive $|OF_2|$ as follows: by Snell's law, $\beta = 36°$; using the fact that the outcoming ray (extended to reach F_2) is perpendicular to o and $|\angle OSC| = 2(180° - 2\beta) + \alpha = 243°$, so C lies in the same quadrant as in the sketch, we have

$|SF_2| = |BS| \sin\alpha = 0.45|BS| = 0.45R$, and $|OF_2| = R + |SF_2| = 1.45R$. The 2nd point source for the interference pattern is F_2, as the rays look like coming from F_2.

Now, what we see in the picture is a double slit experiment, with distance of slits $(1.45 - 0.29)R = 1.16R$. By measuring distances between neighboring intensity minima relative to the image size in photos 2-5, we obtain that to be (on average) approx. 0.12.

Let's utilize the fact that rays tilted by an angle θ from the optical axis o of the lens cross the focal plane of the lens at a distance $f\theta$ from o.

The angular spacing between θ of pairs of rays corresponding to neighboring minima (or maxima) is known to be $\Delta\theta = \frac{\lambda}{1.16R}$ (the distance between slits is 1.3R), and the corresponding distance in the focal plane is $\Delta y = f\Delta\theta = \frac{f\lambda}{1.16R}$.

Now, let's approximate the bubble (which is probably much smaller than the lens) by a single point source. 2 rays incident on the lens can only have θ differ by as much as $\theta_m = \frac{D}{L}$ (near the lens, they can only be as much as D apart; using $D \ll L$ approximation). The image of the bubble in the focal plane, which is formed only by such rays, then has diameter $y = f\theta_m = \frac{fD}{L}$.

The image we obtain is just a magnified version of the one in the focal plane, so in it, the relative distance between neighboring minima $\frac{\Delta y}{y}$ remains the same; from that, we have

$$\frac{\Delta y}{y} = 0.12 = \frac{\lambda L}{1.16RD}$$

$$R = 29\mu m$$

and the bubble diameter is then approximately 58 μm.