## Problem 8: gas bubble in water (contributed by Mihkel Kree)

Introduction. People living in colder climates have surely noticed that by filling a glass with cold tap water one gets a glass of misty (or rather milky) water. The reason is that depressurizing and warming of the water causes the initially dissolved gas to come out of the solution and form tiny bubbles. In this problem you are going to calculate the size of such gas bubble in water.

A photographer prepared a setup consisting of a rectangular water tank with glass walls, a laser beam entering the water tank perpendicularly to one of its faces, and a camera looking directly towards a neighbouring face of the water tank. A gas bubbled entered the laser beam and the photographer managed to take five photos of the bubble while continuously defocusing the camera. The lens had "internal focusing" design, so that defocusing meant changing the focal length while keeping the position of the lens intact, see figure. The line of sight from the camera to the bubble was perpendicular to the laser beam, and the bubble was entirely inside the beam.


In the figure below, the taken photos are placed side by side and indicated by numbers $1-5$.
Task: calculate the diameter of the gas bubble.
Parameters: index of refraction of water with respect to gas: $n=1.3$; wavelength of the laser: $\lambda=488 \mathrm{~nm}$; the lens of the camera can be considered as a single convex lens with focal length $f=10 \mathrm{~cm}$ and diameter $D=3.6 \mathrm{~cm}$ (the change of the focal length due to defocusing was less than $10 \%$ ); the distance from the bubble to the lens: $L=30 \mathrm{~cm}$ (more precisely, this is the distance from the lens to the image of the bubble as seen from the centre of the lens, see figure above).


