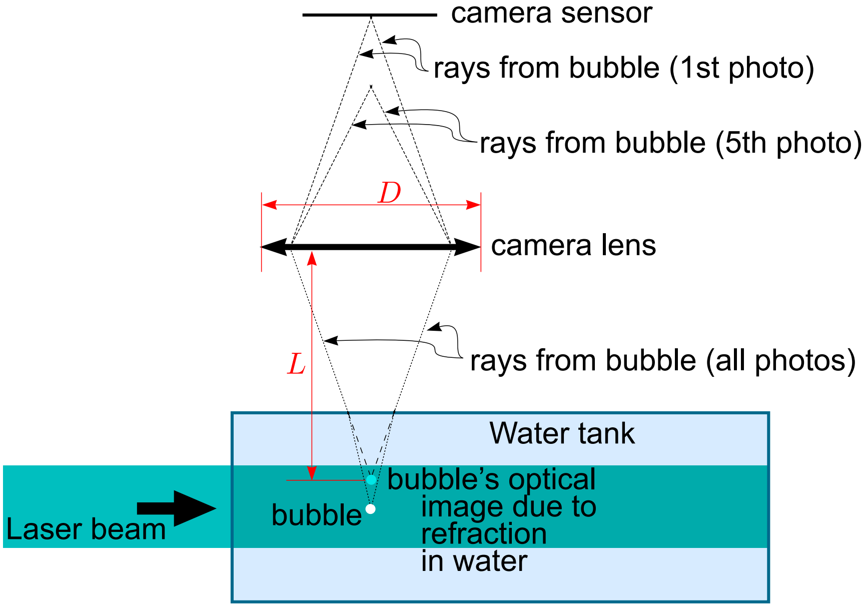
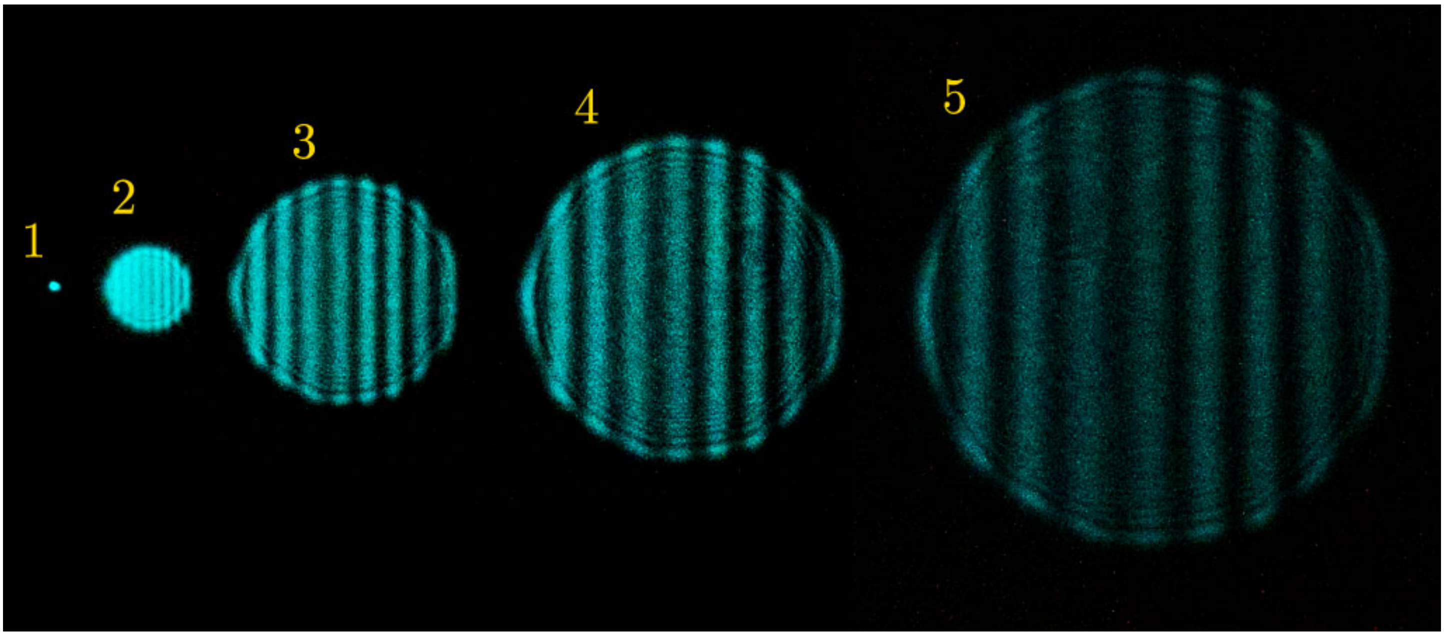
## Problem 8: gas bubble in water (contributed by Mihkel Kree)

**Introduction.** People living in colder climates have surely noticed that by filling a glass with cold tap water one gets a glass of misty (or rather milky) water. The reason is that depressurizing and warming of the water causes the initially dissolved gas to come out of the solution and form tiny bubbles. In this problem you are going to calculate the size of such gas bubble in water.

A photographer prepared a setup consisting of a rectangular water tank with glass walls, a laser beam entering the water tank perpendicularly to one of its faces, and a camera looking directly towards a neighbouring face of the water tank. A gas bubbled entered the laser beam and the photographer managed to take five photos of the bubble while continuously defocusing the camera. The lens had "internal focusing" design, so that defocusing meant changing the focal length while keeping the position of the lens intact, see figure. The line of sight from the camera to the bubble was perpendicular to the laser beam, and the bubble was entirely inside the beam.




In the figure below, the taken photos are placed side by side and indicated by numbers 1 − 5.
**Task:** calculate the diameter of the gas bubble.
**Parameters:** index of refraction of water with respect to gas: $n = 1.3$; wavelength of the laser: $\lambda = 488$ nm; the lens of the camera can be considered as a single convex lens with focal length $f = 10$ cm and diameter $D = 3.6$ cm (the change of the focal length due to defocusing was less than 10%); the distance from the bubble to the lens: $L = 30$ cm (more precisely, this is the distance from the lens to the image of the bubble as seen from the centre of the lens, see figure above).




**Hints after the first week.** If you have a glass ball, observe, what you can see when it is illuminated by a point source (a lamp) from a side. Alternatively, you can study the photo at http://en.wikipedia.org/wiki/File:Clayton_Anderson_zero_g.jpg: (from where the light comes from?).
**Hints after the second week.** We can observe here a nice system of regularly periodic diffraction stripes. Such a diffraction pattern can be observed for two-slit diffraction, but there are clearly no slits in the case of this experimental setup. However, a similar pattern can be observed if there is an interference of light rays coming from two coherent point sources, assuming that the size of the screen on which we observe the interference pattern is much smaller than its distance to the point sources. *(Indeed, if there*

*is a cylindrical screen, and the two point sources are at the axis of the cylinder, the diffraction pattern on the screen will be exactly the same as on a flat screen behind two-slits; if we straighten a small piece of a cylindrical screen the change of its shape is small, and hence, the change of the diffraction pattern on it is also small.)* So, we can make an hypothesis that the diffraction pattern is due to two point sources which are created by the light scattering effects of the bubble. Since the diffraction pattern is very clean (minima are very dark), one can conclude that almost all laser light reaching the lens comes exclusively from those two point sources, and that they have nearly equal brightness.

**Hints after the third week.** There are three apparent candidates for the point sources responsible for the interference pattern. These are images of the laser created (a) via a reflection from the convex bubble surface (which works as a convex mirror, except that the surface is only partially reflecting); (b) via a reflection from the concave bubble surface (in which case the laser light refracts into the bubble, is reflected by the concave surface, and refracts back to the water); (c) via two sequential refractions from the air-water interface of the bubble. Closer inspection shows that via two sequential refractions, the light cannot be diverted as much as by 90 degrees (this would require a larger value of the refraction index). Please pay attention that the bubble cannot be considered as an ideal lens: you can find (and make use of) an image for a narrow beam of light hitting the bubble with an impact parameter $a$, but the position of the image (and the effective focal length) is a function of $a$. Finally, please note that for this problem it may happen that you obtain an equation which needs to be solved numerically.

**Correct results have been submitted by:**

1. Oliver Edtmair (Austria)
2. Kuo Pei-Cheng (Taiwan)
3. David Stein (Germany)
4. Dan-Cristian Andronic (Romania)