

Homework 3

Functional Programming (ITI0212)

Deadline: Wednesday 14th May 2025

Task 1

A decidable proposition is one which is definitively true or false: it is a proposition equipped with the extra data of a concrete witness or counterexample, either $h : p$ or $h : \neg p$.

```
inductive Decidable (p : Prop) where
  | isFalse (h : ¬ p) : Decidable p
  | isTrue (h : p) : Decidable p
```

This definition is part of the Lean standard library.

Prove that

- If propositions p and q are `Decidable`, then so is their conjunction $p \wedge q$
- If propositions p and q are `Decidable`, then so is their disjunction $p \vee q$.
- If a proposition p is `Decidable` then so is its negation $\neg p$.
- If a proposition p is `Decidable` then $p \vee \neg p$ is true.

Task 2

Write functions,

```
sum_to_sigma : ( $\alpha \oplus \beta$ )  $\rightarrow$  ( $\Sigma$  (b : Bool), (if b then  $\alpha$  else  $\beta$ ))
```

```
sigma_to_sum : ( $\Sigma$  (b : Bool), (if b then  $\alpha$  else  $\beta$ ))  $\rightarrow$  ( $\alpha \oplus \beta$ ),
```

that are mutually inverse, i.e. `sum_to_sigma (sigma_to_sum s) = s` and vice-versa.

Optional extra: prove that your functions are mutually inverse.

Task 3

Prove that $\forall (xs : \text{List } \alpha), \text{List.map id } xs = xs$.

Hint: the `congrArg` function may be useful.