Adhesivity, Bigraphs and Bisimulation Congruences

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Abstract. This paper is intended as a short informal summary of some of the topics which arose at the Dagstuhl meeting held 6/06/04-11/06/04. In particular, we shall summarise some of the content of talks by H. Ehrig [1], F. Gadducci [2], O. H. Jensen [3], R. Milner [4], B. König [5], V. Sassone [6] and the author [7]. The general areas include adhesive categories and generalisations, contextual labelled transition semantics for graph transformation systems via borrowed-contexts and GIPOs, and bigraphs. We shall conclude with a summary of some of the discussions which followed the aforementioned presentations.

1 Introduction

This article is an attempt to provide a brief summary of some of the wide range of topics presented and discussed at the Dagstuhl Seminar 04241 "Graph Transformations and Process Algebras for Modeling Distributed and Mobile Systems". We shall concentrate on the notions of adhesivity and reactive systems (in the sense of Leifer and Milner [8]) and examine some of the implications for the well-established theory of double-pushout (DPO) graph transformation [9, 10] and the relatively recently introduced theory of bigraphs [11].

The theory of DPO graph transformation was introduced in the early Seventies [9]. A line of research, the start of which can be attributed to Gadducci and Heckel [12], has recently resulted in the introduction of process algebraic notions, such as context, labelled transition systems and bisimilarity to the field [13, 14]. These latest developments also share the ancestry of Leifer and Milner's approach [8] of deriving canonical labelled transition systems.

Another related line of recent work is the notion of adhesive categories and weaker variations [15–17], which allow a simple categorical "universe" in which one can perform double pushout rewriting and in which classical results such as local Church-Rosser, the concurrency theorem and the parallelism theorems, hold.

2 Adhesive, quasiadhesive and adhesive high-level replacement categories

An *adhesive* category is one where, roughly, pushouts along monomorphisms are "well-behaved", where the paradigm for behaviour is given by the category of

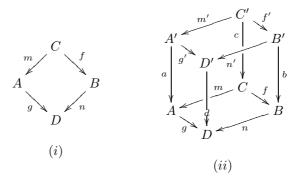
sets. The idea is analogous to that of extensive categories [18], which have wellbehaved coproducts in a similar sense. Various notions of graphical structures used in computer science form adhesive categories. Adhesive categories were introduced in [19].

The notion of adhesivity is too strong for several relevant examples. These examples motivate the study of *quasiadhesive* categories. Roughly, instead of focusing on the behaviour of pushouts along arbitrary monomorphisms, quasiadhesive categories restrict attention to pushouts along *regular* monomorphisms. Quasiadhesive categories, foreshadowed in [19], are introduced in [16]. Regular monomorphisms are, in general, a proper subclass of the class of monomorphisms. One can parametrise the definition of adhesivity with respect to an arbitrary class of monomorphisms, provided that the class satisfies certain axioms. This approach yields the adhesive HLR categories, introduced in [17].

Here we shall recall the definitions of adhesive, quasiadhesive and adhesive HLR categories as well as some of their properties. The three classes depend on the notion of van Kampen squares, which we introduce below.

2.1 Van Kampen squares

Definition 1 (van Kampen square). A van Kampen (VK) square (i) is a pushout which satisfies the following condition:



given a commutative cube (ii) of which (i) forms the bottom face and the back faces are pullbacks, the front faces are pullbacks if and only if the top face is a pushout. Another way of stating the "only if" condition is that such a pushout is required to be stable under pullback.

Lemma 1. In a VK square as in (i), if m is a monomorphism then n is a monomorphism and the square is also a pullback.

2.2 Adhesive categories

Definition 2 (Adhesive category). A category C is said to be *adhesive* if

(i) **C** has pushouts along monomorphisms;

(ii) **C** has pullbacks;

(iii) pushouts along monomorphisms are VK-squares.

Example 1. Set is adhesive.

Example 2. The categories Pos, Top, Gpd and Cat are not adhesive.

Proposition 1.

- (i) If **C** and **D** are adhesive categories then so is $\mathbf{C} \times \mathbf{D}$;
- (ii) If C is adhesive then so are C/C and C/C for any object C of C;

(iii) If \mathbf{C} is adhesive then so is any functor category $[\mathbf{X}, \mathbf{C}]$.

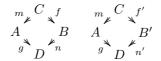
The closure conditions outlined in Proposition 1 ensure that categories of ordinary directed graphs, typed graphs [20] and hypergraphs [21], amongst others, are adhesive categories.

The following two lemmas are used extensively in literature on graph transformation.

Lemma 2. Monomorphisms are stable under pushout.

Lemma 3. Pushouts along monomorphisms are also pullbacks.

Lemma 4. Pushout complements of monos (if they exist) are unique up to isomorphism. In other words, given two pushouts along mono m as illustrated below, there exists an isomorphism $\varphi: B \to B'$ such that $n'\varphi = n$ and $\varphi f = f'$.



Lemma 5. Monomorphisms are regular.

2.3 Quasiadhesive categories

For certain applications, including the theory of algebraic specifications, the axioms of adhesive categories are too restrictive. Indeed many examples fail to be adhesive, but are quasiadhesive. In this brief summary, we shall recall the definition of quasiadhesive categories and present some of their theory. The reader is directed to [16] for a more complete presentation.

Definition 3. A category C is said to be quasiadhesive if

- (i) **C** has pushouts along regular monomorphisms;
- (ii) **C** has pullbacks;
- (iii) pushouts along regular monomorphisms are VK-squares.

Adhesive categories can be seen as "degenerate" quasiadhesive categories, as explained by the following fact.

Proposition 2. The adhesive categories are precisely the quasiadhesive categories in which every monomorphism is regular.

The following lemma sums up a two important properties of regular monos in quasiadhesive categories.

Lemma 6. The following hold in any quasiadhesive category C:

- (i) regular monomorphisms are stable under pushout;
- (ii) regular monomorphisms are closed under composition.

2.4 Adhesive high-level replacement categories

We have seen the definition of adhesive and quasiadhesive categories, which share the central concept of a van Kampen square. Indeed, in adhesive categories all pushouts along monomorphisms are VK squares, while in quasiadhesive categories it is the pushouts along regular monomorphisms which satisfy this property.

The definition of "adhesivity" can be generalised so as to be parametrised over a class of monomorphisms \mathcal{M} . This was done in [17], resulting in the class of adhesive high level replacement (adhesive HLR) categories. We present the definition below.

Definition 4 (Adhesive HLR category). A category \mathbf{C} with an associated class of monomorphism \mathcal{M} is an adhesive HLR category when:

- $-\mathcal{M}$ is closed under composition: if $f: A \to B$ and $g: B \to C$ are both in \mathcal{M} then $gf: A \to C$ is in \mathcal{M} ;
- \mathcal{M} is closed under decomposition: for any $f : A \to B$, and any $g : B \to C$ in $\mathcal{M}, gf : A \to C$ in \mathcal{M} implies that $f : A \to B$ is in \mathcal{M} ;
- C has pushouts along \mathcal{M} morphisms;
- **C** has pullbacks along \mathcal{M} morphisms;
- $-\mathcal{M}$ morphisms are closed under pushouts;
- $-\mathcal{M}$ morphisms are closed under pullbacks;
- pushouts along \mathcal{M} morphisms are VK-squares.

Clearly, adhesive categories are adhesive HLR categories where \mathcal{M} is the class of all monomorphisms. Similarly, quasiadhesive categories are adhesive HLR categories with \mathcal{M} being the class of regular monomorphisms.

Adhesive HLR categories satisfy closure conditions analogous to the closure conditions specified in Proposition 1.

An important new example of adhesive HLR categories is the category of typed attributed graphs [22].

3 Applications to DPO Transformation

HLR categories [21] were originally introduced in order to generalise the theory of DPO graph transformation systems to a suitable class of categories. Adhesive, quasiadhesive and adhesive HLR categories can be seen as elegant replacements for HLR categories; indeed, many of the axioms assumed in the theory of HLR categories follow as simple lemmas in these categories. In [19] this was demonstrated explicitly with the proof of the local Church-Rosser theorem and the concurrency theorem in the setting of DPO systems over adhesive categories. Ehrig et al. [17] extended the rich rewriting theory further by considering the important new concepts of initial pushouts and critical pairs in the setting of DPO systems over adhesive HLR categories.

4 Deriving bisimulation congruences

An important new development discussed extensively at the seminar has been the application of Leifer and Milner's theory of reactive systems and relative pushouts (RPOs) [8] to DPO graph transformation systems. This work been done independently by Ehrig and König [13] and by Sassone and Sobociński [14].

Both approaches define a "graph context" to be a cospan of graph morphisms. Intuitively, a graph is assigned an input interface and an output interface. A graph with an output interface can be substituted into a context if the input interface of the context agrees with the output interface of the graph, the composition is then performed via the pushout construction.

This "contextual interpretation" is useful when graphs are used to encode a concurrent agent – indeed, in such situations it is not enough to study the behaviour of the agent itself; it is necessary to experiment with the agent by substituting it into contexts and observing the behaviour of the resulting systems. This, however, brings up the problem of how to reason about such "agents", in particular, when can two graphs be considered to be equivalent. Ehrig and König, motivated by the approach of Leifer and Milner, derive a labelled transition system (lts) with labels being the so-called borrowed contexts. Moreover, they prove that bisimilarity on the resulting lts is a congruence with respect to all graph contexts. The intuition is the same as for Leifer and Milner, that is, borrowed contexts are intuitively the smallest graph-contexts which allow interaction, in the sense that the composition allows a rewrite to be performed.

Indeed, one can consider DPO systems as reactive systems over cospan categories, in sense of Leifer and Milner, in a very natural way [12, 14]. Conversely, any reactive system over a cospan category can be considered as a certain "contextual" DPO rewriting system. It turns out that GRPOs can be constructed in certain such cospan categories, and that the notion of borrowed context coincides with GIPOs in such categories.

4.1 Borrowed contexts

To allow a graph to be substituted into an arbitrary graph context, one enriches it with an interface; substitution is defined when the interface matches the input interface of the context and the substitution is performed by pushout. All graph morphisms in this section are assumed to be injective.

Definition 5. A graph with an interface is simply a graph G together with a homomorphism $J \to G$. A graph context is a cospan of graph morphisms $J \to F \leftarrow K$. The composition is performed by pushout, as illustrated below.

$$J \longrightarrow F \longleftarrow I$$
$$\downarrow \qquad \qquad \downarrow$$
$$G \longrightarrow G^+$$

The resulting graph G^+ has an output interface K obtained by composition of the two rightmost morphisms in the above diagram.

In order to answer the question of what it means, operationally, for a context to allow a graph to rewrite, consider the diagram below.

$$D \longrightarrow L \stackrel{l}{\leftarrow} I \stackrel{r}{\longrightarrow} R$$

$$\downarrow (1) \stackrel{\ell}{\downarrow} (2) \stackrel{l}{\downarrow} (3) \stackrel{\ell}{\downarrow} \theta$$

$$G \stackrel{-\epsilon_1}{\rightarrow} G^+ \stackrel{\epsilon_2}{\leftarrow} 0^2 - C \stackrel{-\theta_2}{\rightarrow} H$$

$$o_G \stackrel{l}{\downarrow} (4) \stackrel{\ell}{\to} 1 \quad (5) \quad \uparrow o_C$$

$$J \stackrel{\ell}{\longrightarrow} F \stackrel{\epsilon_F}{\longleftarrow} F \stackrel{\epsilon_{OF}}{\longleftarrow} K$$

$$(i)$$

Starting with a graph G with an interface $o_G : J \to G$, we compose with a context $J \xrightarrow{\iota_F} F \xleftarrow{o_F} K$ and obtain a graph G^+ by constructing the pushout (4). The resulting graph G^+ allows a double-pushout rewrite – that is, starting with a rewrite rule $L \xleftarrow{l} I \xrightarrow{r} R$, we are able to construct pushouts (2) and (3) to obtain a graph H. The basic idea of borrowed contexts, inherited from the theory of relative pushouts, is that one constructs a labelled transition system using the "smallest" contexts which allow reaction as labels. We recall give the definition of the lts below.

Definition 6 (Rewriting with borrowed contexts). Assuming a fixed set \mathcal{P} of DPO productions (spans $L \xleftarrow{l} I \xrightarrow{r} R$), define a labelled transition system as follows

- states: graphs with output interfaces $J \rightarrow G$;
- transitions: there is a transition from $J \to G$ to $K \to H$ with label $J \to F \leftarrow K$ precisely when there exists a DPO production $L \leftarrow I \to R$ so that regions (4), (2) and (3) are pushouts (the context allows a rewrite), and additionally, (5) is a pullback and there exists an object D with morphisms $D \to G$ and $D \to L$ making (1) a pushout.

A central result of [13] is the following.

Theorem 1. Bisimilarity on the resulting lts is a congruence with respect to all graph contexts.

4.2 GIPOs and cospans

Here we shall recall the main results of [14] and expanded in [23]. The original framework of reactive systems and relative pushouts [8] can be extended smoothly to a 2-dimensional setting, so that the underlying category of a reactive system is a 2-category or a bicategory. The notions of relative pushouts (RPO) and idem-relative pushouts generalise to groupoidal-relative pushouts (GRPO) and groupoidal-idem-relative pushout (GIPO) [24]. Categorically, a GRPO is a bipushout in a pseudo-slice category. Roughly, a GIPO is a diagram which results from the construction of a GRPO. As IPOs characterise the smallest contexts which allow reaction in a categorical setting, so GIPOs characterise such contexts in a 2-categorical setting. The good behaviour of the labelled transition systems generated using Leifer and Milner's theory lifts to the 2-categorical setting, in particular, one obtains congruence theorems for bisimilarity as well as other equivalences, see [24, 23] for details.

Given an adhesive category \mathbf{C} , we let $\mathrm{ILC}(\mathbf{C})$ denote the bicategory of *inputlinear* cospans, that is cospans $I_1 \stackrel{\iota}{\longrightarrow} C \stackrel{o}{\longleftarrow} I_2$ where ι is mono. The following theorem is the main result of [14].

Theorem 2. For any adhesive category \mathbf{C} , $ILC(\mathbf{C})$ has GRPOs.

There is a very close correspondence between GIPOs and borrowed contexts. Indeed, if we restrict to *linear* cospans (both arrows of the cospan are mono), we are in a position to compare the two approaches as the theory of borrowed contexts requires the assumption of all morphisms being mono. The proof of the following theorem can be found in [14, 23].

Theorem 3. The (concrete) labelled transition system generated using GIPOs is equal to the labelled transition system generated using borrowed contexts.

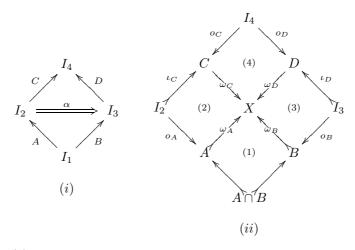
As a consequence, one may view Theorem 1 as a corollary of the standard congruence theorem for reactive systems. One also gets the congruence theorems for other equivalences, such as failures equivalence and trace equivalence.

The existence of GIPOs in $ILC(\mathbf{C})$ also allows one to extend the notion of borrowed context, allowing non-injective output interfaces. This is presented in detail in [14].

It turns out that the definition of borrowed contexts can be used to characterise GIPOs in this general setting.

Proposition 3. Diagram (i) is a GIPO if and only if there exists an X and isomorphisms $\omega_l : A +_{I_2} C \to X$ and $\omega_r : B +_{I_3} D \to X$ such that $\omega_r^{-1} \omega_l = \alpha$

and so that in the resulting diagram (ii) we have:



- region (1) is both a pullback and a pushout;
- regions (2) and (3) are pushouts;
- region (4) is a pullback.

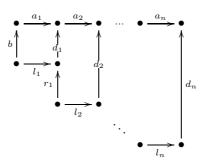
It is this characterisation which is at the heart of Theorem 3 and which allows us to relate borrowed contexts and GIPOs – in particular, it shows that borrowed contexts arise from a general categorical categorical construction and thus satisfy a universal property.

5 Bigraphs and weak bisimilarity

Here we give a brief overview of the presentation by Jensen [3]. The results shall be published in Jensen's upcoming PhD thesis [25].

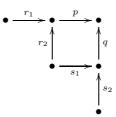
The main idea is to extend the behavioural theory of bigraphs by allowing a weak version of bisimilarity – that is, one which abstracts the internal behaviour of agents and focuses solely on interactions with the environment. The results presented are not specialised to bigraphs and generalise nicely to *any* reactive system in the sense of Leifer and Milner, and therefore, we shall recall them in that setting.

The inspiration for weak bisimilarity comes from traditional labelled transition systems in the field of process calculus which come equipped with a special label τ – the intuition is that a τ labelled transition corresponds to a reaction within the term with no outside interaction needed from the environment. Supposing that the labels of the lts come from some set A, we can define, for an arbitrary string $a = a_1 \dots a_n$ in L^* , a "weak" labelled transition $b \stackrel{a}{\Rightarrow} b'$ to mean a sequence of transitions in which falls into the family $\tau^* a_1 \tau^* \dots \tau^* a_n \tau^*$. One then defines weak bisimilarity and other weak equivalences by using such weak transitions. Roughly, technically we are saturating the lts with τ labels, making it impossible to distinguish two processes by their internal behaviour. A similar process can be repeated in the setting of reactive systems.



Jensen proposes to consider a new lts defined as follows: $b^{\underline{a_n...a_1}} \flat b'$ if, for $1 \leq n \leq n$, there exists a sequence of reaction rules $\langle l_i, r_i \rangle \in \mathcal{R}$ and reactive contexts d_i so that $b_i \xrightarrow{a_i} \flat_{i+1}$ in the ordinary transition system, $b_i = d_i r_i, b \xrightarrow{a_1} \flat_1$ and $b' = b_n$. The resulting diagram of IPOs is illustrated above. Jensen also argues that it is important to require that a_2, \ldots, a_n are reactive contexts.

The congruence theorems can be proved in a particularly simple and attractive way: consider two reactive systems over a category **C** with reaction rules \mathcal{R} and \mathcal{S} . One can define \mathcal{RS} as the set of reaction rules obtained by "composing" the reaction relations as follows: given $\langle r_1, r_2 \rangle \in \mathcal{R}$ and $\langle s_1, s_2 \rangle \in \mathcal{S}$, their composition is a family of reaction rules $\langle pr_1, qs_2 \rangle \in \mathcal{RS}$, where p and q are required to be reactive and the square below is required to be an IPO.



Letting ϵ be the reaction relation consisting of the single rule {id, id}, one defines $\mathcal{R}^* = \epsilon \cup \mathcal{R} \cup \mathcal{R} \mathcal{R} \cup \ldots$ It is now easy to check that Jensen's weak lts is just the "ordinary" lts over the new reactive system with \mathcal{R}^* as its reaction relation. In particular, this implies the congruence theorems.

When stated in the generality of reactive systems, the results of Jensen's formulation apply to bigraphs as well as borrowed contexts – using the translation of double pushout transformation systems into reactive systems over cospan bicategories.

6 Bigraphs and graphical formalisms for calculi

In a series of talks, Milner and Jensen [4] presented some of the theory of bigraphs. Bigraphs are a graphical system which falls within the framework of Leifer and Milner's reactive systems and has an associated behavioural theory induced by RPOs. Their purpose is to bring together elements of foundational calculi for concurrency and mobility, such as the Pi-calculus and the calculus of mobile ambients. One of the advantages of a graphical presentation over a syntactic presentation is that one often avoids or greatly simplifies structural congruence relations, which are often largely subsumed by graph isomorphisms.

One of the chief features of bigraphs is the orthogonal treatment of a tree topographical structure – often used to model physical structure – and a logical link structure. Roughly, one may say that "where you are doesn't affect with whom you may talk to".

Certain bigraphs can be considered as particular cospan bicategories. Thus, one may to some extent apply Theorem 2 in order to derive the existence of GRPOs; meaning that Milner's construction of RPOs for bigraphs can be seen as a special case of a more general construction. This is, however, not entirely satisfactory as it appears that a construction of GRPOs for output-linear cospans would be more appropriate to deal with some of the phenomena of bigraphs. Another advantage of defining bigraphs as cospans is that their reactions can be seen as DPO transformations – meaning that aspects of the theory of DPO transformation systems can be applied to bigraphs.

In his presentation [2], Gadducci presented a graphical formalism in which one may encode the pi-calculus [26, 27]. As opposed to bigraphs, Gadducci's encoding uses only ordinary (typed) graphs. It may prove interesting as future work to ascertain to what extent one may use the technology of reactive systems and (G)RPOs in order to analyse the behavioural theory of pi by studying this encoding.

7 Conclusions

We have discussed aspects of adhesive, quasiadhesive and adhesive HLR categories and their relationship to the field of double-pushout transformation systems. Moreover, we have compared the borrowed-context approach and the GIPO approach of endowing such a system with a compositional labelled transition system semantics. We have briefly discussed a way of defining weak equivalences in the setting of reactive systems with application to bigraphs, but also to any reactive systems and in particular to "contextual" DPO transformation systems, since these are reactive systems over cospan bicategories.

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