ITT8040 CELLULAR AUTOMATA
EXAM SHEET – QUESTIONNAIRE

May 22, 2013

Name: ____________________________

1. State the compactness principle:
Every sequence of configurations has a convergent subsequence.

2. State the definition of pre-injectivity:
A cellular automaton with global function G is pre-injective if, for every pair of configurations c, e such that c and e are not equal but differ only on finitely many points, we have $G(c) \neq G(e)$.

3. State the Garden-of-Eden theorem:
A cellular automaton is surjective if and only if it is pre-injective.

4. Construct the look-up tables of the following elementary cellular automata. Which of them are injective? Which ones are surjective? Write “Yes” or “No” in the corresponding box.

	Injective	Surjective
102 *f(x,y,x) = y XOR z*	No	Yes
136 *f(x,y,z) = y AND z*	No	No
170 *f(x,y,z) = z: is the shift*	Yes	Yes
236 *is not balanced*	No	No

(x,y,z)	102	136	170	236
0,0,0	0	0	0	0
0,0,1	1	0	1	0
0,1,0	1	0	0	1
0,1,1	0	1	1	1
1,0,0	0	0	0	0
1,0,1	1	0	1	1
1,1,0	1	0	0	1
1,1,1	0	1	1	1

5. Is Conway's Game of Life surjective?
____ Yes
_X__ No

Motivate your answer: *Any configuration with a single living cell becomes quiescent in one step, so that Conway's Game of Life is not injective on finite configurations: by the Garden-of-Eden theorem, it is not surjective.*

6. Let A be a one-dimensional cellular automaton with set of states $S = \{0,1\}$, neighborhood $N = \{n_1=0, n_2, \ldots, n_m\}$, and local update rule f. Mark with an X those properties from the following list that are ***equivalent*** to the surjectivity of A:

_X__ A is surjective on periodic configurations.

____ A is injective on periodic configurations. *This is equivalent to reversibility.*

____ Every pattern has 2^{m-1} pre-images. *Not the same if support is not an interval.*

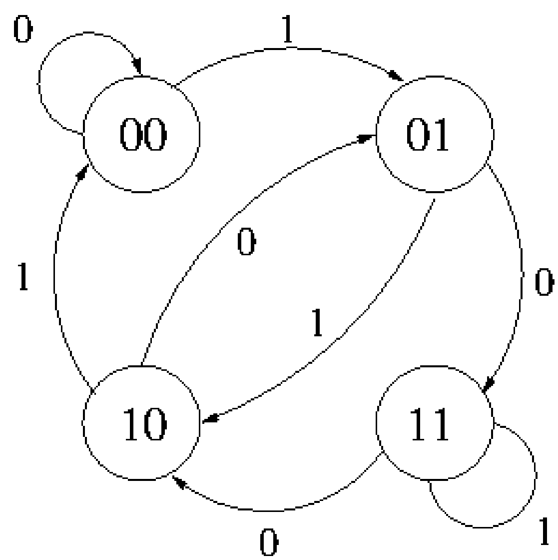
____ The look-up table of f is balanced. *Not sufficient, cf. the majority rule.*

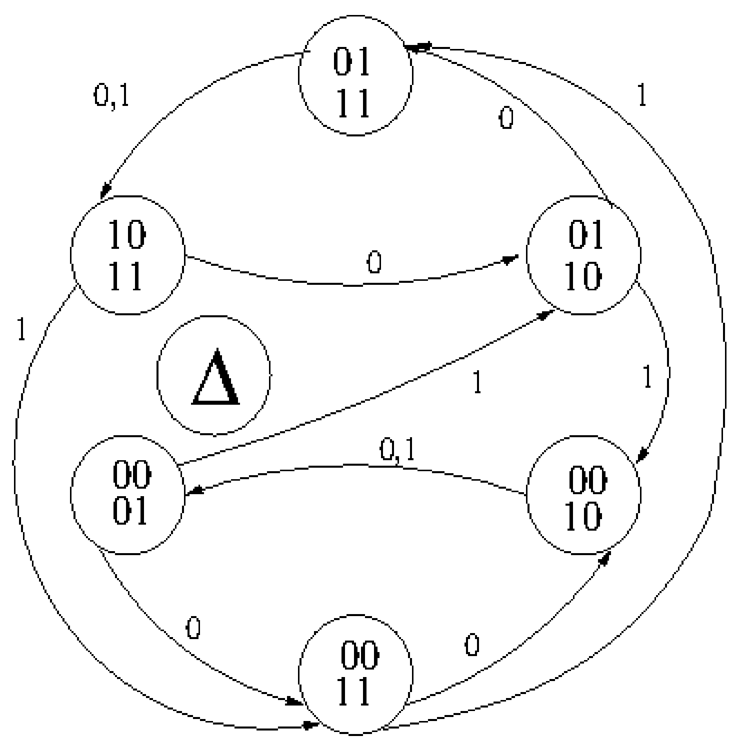
_X__ No configuration has uncountably many pre-images.

_X__ Each configuration has strictly less than M pre-images, for some M.

7. Construct the labeled de Bruijn graph of elementary cellular automaton rule 150.

Bonus: construct the reduced pair graph.





8. Which of the following problems are semi-decidable? Which ones have a semi-decidable complement? Write “Yes” or “No” in the corresponding box.

	R.e.	Co-r.e.
Finite tiling problem	Yes	No
Periodic tiling problem	Yes	No
Turing's halting problem	Yes	No
Surjectivity of 2D cellular automata	No	Yes

9. Let $g : \mathbb{N} \to \mathbb{N}$ be a function such that every 2D reversible CA defined by a rule on a Moore neighborhood of radius n has an inverse CA which is defined by a rule on a Moore neighborhood of radius at most $g(n)$. Can g be computable?

____ Yes

_X__ No

Motivate your answer: *If g was computable, we would be able to decide reversibility of any 2D CA by first considering n such that the CA is defined by a local rule on a Moore neighborhood of radius n, then compute R = g(n), and check, for every CA defined on the Moore neighborhood of radius R, whether it yields the identity when composed with the initial CA. But this would go against Kari's undecidability theorem.*

10. A cellular automaton with the Margolus neighborhood may be:

	YES	NO
Injective and surjective	X	
Surjective, but not injective		X
Neither surjective nor injective	X	
Injective, but not surjective		X

11. Suppose that a cellular automaton $A = (S,d,N,f)$ with quiescent state q has a nontrivial conserved quantity, induced by an additive quantity $\mu : S \rightarrow \mathbb{R}$. (Recall that we may suppose $\mu(q) = 0$.) Can such CA be nilpotent?

_____ Yes

_X__ No

Motivate your answer: *As the conserved quantity is nontrivial, there exist a finite configuration c where its value is nonzero. As the value of the conserved quantity on the quiescent configuration is zero, c can never become quiescent, no matter how many times the CA is iterated.*