MIDTERM EXAM

ITT 9130 Concrete mathematics

November 11, 2014

1. Prove that, for every nonnegative integer n, the quantity

$$n^{16} - n^{14} - n^4 + n^2$$

is divisible by 78.

2. Solve the following recurrence:

$$\begin{aligned} T_0 &= 1\,, \\ T_n &= -nT_{n-1} + 3 \cdot n \cdot n! \quad \text{for } n > 0\,. \end{aligned}$$

Hint: consider the general solution to the recurrence:

$$\begin{aligned} U_0 &= \alpha\,, \\ U_n &= U_{n-1} + (-1)^n \cdot (\beta n + \gamma) \quad \text{for } n > 0\,. \end{aligned}$$

3. For $n \geq 0$, evaluate

$$S_n = \sum_{0 \leq k < n} k(k-1)H_k\,.$$