Tallinn University of Technology
TTÜ School of Information Technology

ITT9132 Concrete Mathematics

Extended syllabus
Spring 2019

Course aims/objectives: To provide students with mathematical tools for the study of recurrence equations and their applications to relevant to computing and information technology.

Learning outcomes: At the end of the course, the successful students:

1. Know the fundamental concepts and methods of continuous and discrete mathematics relevant to computing and information technology.
2. Solve problems of discrete mathematics with the aid of methods from continuous mathematics.
3. Apply mathematical methods to the analysis of algorithms.

Brief description of the course (topics): The course "Concrete Mathematics" deals with topics in both continuous and discrete mathematics (whence the name) with a special focus on the solution of recurrence equations. It is aimed at Master and Doctoral students of the School of Information Technology.

The course will discuss several topics which have important applications in advanced computer programming and the analysis of algorithms. Selection is made from the following topics according to interests and preliminary knowledge of students:

1. Sums. Sums and recurrences. Manipulation of sums. Multiple Sums. General methods of summation. Finite and Infinite calculus. Infinite sums.
2. Integer Functions. Floors and ceilings. Floor/Ceiling applications. Floor/Ceiling recurrences. Floor/Ceiling sums.
3. Number Theory. Divisibility. Prime numbers. Greatest common divisor. Primality testing. The Euler and Möbius functions.
4. Binomial Coefficients. Basic Identities. Applications. Generating functions for binomial coefficients.
5. Special Numbers. Stirling numbers of the second and of the first kind. Fibonacci numbers. Harmonic numbers. Bernoulli numbers.
6. Generating Functions. Basic maneuvers. Solving recurrences. Convolutions. Exponential generating functions.
7. Discrete Probability. Mean and variance. Probability generating functions. Flipping coins. Hashing.
8. Asymptotics. Big-O notation. Big-O manipulation. Bootstrapping. Trading tails. Euler's summation formula.

Language of the course: English

ECTS credits: 6 ECTS

Students: This course is addressed mainly to Master and Doctoral students of the School of Information Technology.

Special needs: Persons with disabilities can participate in this course. Please inform the instructor(s) as soon as possible regarding any special instruction, or assessments of this course that may be necessary to enable you to fully participate in this course.

Registration: Students who would like to take the course should declare the course in the ÕIS (Student Information System) by the deadlines set in the academic calendar.

Prerequisite courses and/or knowledge: First-year university level of algebra and calculus, plus the basics of combinatorics (Newton's binomial theorem). Such prerequisites can be provided, for example, by IAX0010 Discrete Mathematics, or by ITT0030 Discrete Mathematics II.

Prerequisite resources: Textbook, pen, paper, time, and energy. Software is not required, but can be used for study if helpful. For free student download of MS Office see the instructions at https://wiki.ttu.ee/it/en/doc/office

Professor(s): Silvio Capobianco, Department of Software Science, School of Information Technology. silvio.capobianco@taltech.ee

Contacting Professor(s): Preferred means of contact: email; responses provided within 2 workdays.

Schedule for classes: 16 weeks, 2 hours/week lecture, 2 hours/week exercises.

Study process description: Ideally, each week, the students will, in this order:

1. Attend the lecture.
2. Study the textbook material covered in the lecture.
3. Attempt the exercises related to those topics.
4. Participate in the exercise session.
5. Review the material discussed during the week.
6. Compile personal notes.

Course's e-support: Course materials can be accessed via the course web page at the address http://www.cs.ioc.ee/~silvio/2019/ITT9132/index.html

Study literature: Textbook:

- Ronald Graham, Donald Knuth, and Oren Patashnik. Concrete Mathematics: a Foundation for Computer Science. Addison-Wesley 1994.

Continuous assessment:

- Two classroom presentations, each counting for 10% of the final grade.
- One midterm exam, counting for 25% of the final grade.

Evaluation criteria for continuous assessment: For each classroom presentation, each student will give a 10-minutes talk discussing their original solution of an exercise from the book, chosen together with the instructor.
For the midterm exam, the students will solve classroom exercises.
Only handwritten notes are allowed. Electronic devices, with the exception of a pocket or tabletop calculator, are forbidden.

Exam: The final exam will take place at the end of the semester, be given on two dates, and count for 55% of the final grade.
To be admitted to the final exam, a student must have earned 21 attendance tokens, given at least one classroom presentation, and completed at least 50% of the midterm exam. The attendance requirement can be removed in case of serious impediments such as illness or long work transfers.
Student who do not take the final exam, or do not return the assignment, will receive a "no show" mark.
Students can take the final exam on both dates: in this case, the last returned assignment will determine the final grade.

Evaluation criteria for the exam: The students will perform a series of exercises including, but not limited to, problem solving and open-answer questions.
Only handwritten notes are allowed. Electronic devices, with the exception of a pocket or tabletop calculator, are forbidden.

Final grade:	From 5 (maximum) down to 0 (minimum). The sum of points for the two midterms and the final exam is converted into the final grade according to the following conversion table: "5" excellent, 91-100 points "4" very good, 81-90 points "3" good, 71-80 points "2" satisfactory, 61-70 points "1" poor, 51-60 points "0" fail, 50 points or less The final score will reflect the following: 5: The student commands the subject. 4: The student has a good grasp on the subject, with some small mistakes or imprecisions. 3: The student understands most of the subject, but there are some evident major issues. 2: The student manages the bulk of the subject, but also shows serious lacks or misunderstandings. 1: The student achieved the bare minimum. Maybe the approach to the course was flawed. 0: At the end of the course the student did not display an appreciable knowledge of the subject.
Academic integrity:	As a student at TTÜ, you have an obligation to conduct your academic work with honesty and integrity according to University standards. It is expected that all work that you submit will be your own, and that you have actually done the work that you are submitting. Plagiarism and cheating will not be tolerated. Should you be found to be guilty of such activities, it will be followed with grade "0" for the assignment/exam and a notice will be filed to the School's Committee for Handling Violations of Academic Practice and Contemptible Behaviour. Depending on the Committee's proposal, it may lead to Dean issuing a letter of reprimand or in case of repeated or very severe misconduct, exmatriculation from the University.

Detailed schedule and topics

For each week, the lecture will take place on Monday, and the exercise session on Wednesday.
Exceptions are made for the week of the Estonian Winter School in Computer Science and for those dates which fall on national holidays. In those cases, recovery dates are arranged.

The students are to be familiar with the material in each lecture by the time of the exercise session of the same week. (only if lectures and exercises on different days)

Slides from the lectures and solutions to the classroom exercises will be uploaded by the afternoon of the next day.

Week 1 – 28.01.2019-03.02.2019
Introduction to the course.

Week 2 – 04.02.2019-10.02.2019
Recurrent problems. The Tower of Hanoi. Lines in the plane. The Josephus problem.
Textbook: Chapter 1, pages 1-12.

Week 3 – 11.02.2019-17.02.2019
Recurrent problems. Binary representation. Generalization of the Josephus function. The repertoire method.
Sums. Notation. Sums and recurrences. The perturbation method. Summation factors.
Textbook: Chapter 1, pages 12-16; Chapter 2, pages 21-27.

Week 4 – 18.02.2019-24.02.2019
Sums. Summation factors. Efficiency of quicksort. Manipulation of sums. Multiple sums. General methods.
Textbook: Chapter 2, pages 28-46.

Week 5 – 25.02.2019-03.03.2019
Sums. Finite and infinite calculus. Infinite sums.
Integer Functions. Floors and ceilings. Floor/Ceiling applications. Floor/Ceiling recurrences. ``mod'': the binary operation. Floor/Ceiling sums.
Textbook: Chapter 2, pages 47-62; Chapter 3, pages 67-94.

Week 6 – 04.03.2019-10.03.2019
24th Estonian Winter School in Computer Science.

Week 7 – 11.03.2019-17.03.2019
Number Theory. Prime and composite numbers. Divisibility. Greatest common divisor. The Euclidean algorithm. Prime numbers. The Fundamental Theorem of Arithmetics. The distribution of prime numbers.
Textbook: Chapter 4, pages 102-111, 115-123.

Week 8 – 18.03.2019-24.03.2019
Number Theory. Modular arithmetics. Primality tests. Fermat's test. Rabin-Miller test. Euler and Möbius functions.
Textbook: Chapter 4, pages 123-144.

Week 9 – 25.03.2019-31.03.2019
Binomial Coefficients. Basic identities. Basic practice. Generating functions. Intermezzo: Analytic functions.
Textbook: Chapter 5, pages 153-175, 196-197.

Week 10 – 01.04.2019-07.04.2019
Binomial Coefficients. Generating functions. Operations on generating functions. Building generating functions that count. Identities in Pascal's triangle.
Textbook: Chapter 5, pages 196-204.

Week 11 – 08.04.2019-14.04.2019
Special Numbers. Stirling numbers of the first and second kind. Stirling's inversion formula. Generating function of falling and rising factorials. Fibonacci numbers. Generating function of Fibonacci numbers.
Textbook: Chapter 6, pages 257-267, 290-292, 297-299.

Week 12 – 15.04.2019-21.04.2019
Special Numbers. Fibonacci numbers. Cassini's identity. Harmonic numbers. Harmonic summations. Bernoulli numbers.
Textbook: Chapter 6, pages 272-290, 292-297.

Week 13 – 22.04.2019-28.04.2019
Generating Functions. Solving recurrences with generating functions. Partial fraction expansion. The Rational Expansion Theorem.
Textbook: Chapter 7, pages 331-341.

Week 14 – 29.04.2019-05.05.2019
Generating Functions. Use of derivatives. Convolutions. Catalan numbers. Exponential generating functions.
Textbook: Chapter 7, pages 341-369.

Week 15 – 06.05.2019-12.05.2019
Asymptotics. A hierarchy. Big-O notation. Big-O manipulation.
Textbook: Chapter 9, pages 439-457.

Week 16 – 13.05.2019-19.05.2019

Asymptotics. Big-O manipulation. Two asymptotic tricks. Euler's summation formula. Stirling's approximation for the logarithm of the factorial.

Chapter 9, pages 452-453, 463-475, 481-489.

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