ITT9132 Concrete Mathematics
Exercise session 5: 25 February 2021

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Exercise 2.8

What is the value of $0^{\underline{m}}$, when m is a given integer?

Exercise 2.10

The text derives the following formula for the difference of a product:

$$\Delta(uv) = u\Delta v + Ev\Delta u\ . \tag{1}$$

How can this formula be correct, when the left-hand side is symmetric with respect to u and v but the right-hand side is not?

Exercise 2.16

Prove that $x^{\underline{m}}/(x-n)^{\underline{m}} = x^{\underline{n}}/(x-m)^{\underline{n}}$ unless one of the denominators is zero.

Exercise 2.27

Compute $\Delta(c^{\underline{x}})$, and use it to deduce $\sum_{k=1}^{n}(-2)^{\underline{k}}/k$.

Exercise 2.28

At what point does the following derivation go astray?

$$
\begin{aligned}
1 &= \sum_{k\geqslant 1} \frac{1}{k\cdot(k+1)} && (2)\\
&= \sum_{k\geqslant 1} \left(\frac{k}{k+1} - \frac{k-1}{k}\right) && (3)\\
&= \sum_{k\geqslant 1}\sum_{j\geqslant 1} \left(\frac{k}{j}[j=k+1] - \frac{j}{k}[j=k-1]\right) && (4)\\
&= \sum_{j\geqslant 1}\sum_{k\geqslant 1} \left(\frac{k}{j}[j=k+1] - \frac{j}{k}[j=k-1]\right) && (5)\\
&= \sum_{j\geqslant 1}\sum_{k\geqslant 1} \left(\frac{k}{j}[k=j-1] - \frac{j}{k}[k=j+1]\right) && (6)\\
&= \sum_{j\geqslant 1} \left(\frac{j-1}{j} - \frac{j}{j+1}\right) && (7)\\
&= \sum_{j\geqslant 1} \frac{-1}{j\cdot(j+1)} && (8)\\
&= -1 && (9)
\end{aligned}
$$

Exercise from midterm test of 8 November 2016

1. Prove that $H_n \leqslant 1 + \log_2 n$ for every $n \geqslant 1$.
2. Use the previous point to evaluate $\sum_{k=1}^{\infty} k^{-2} H_k$.