Computing Exact Outcomes of Multi-Parameter Attack Trees

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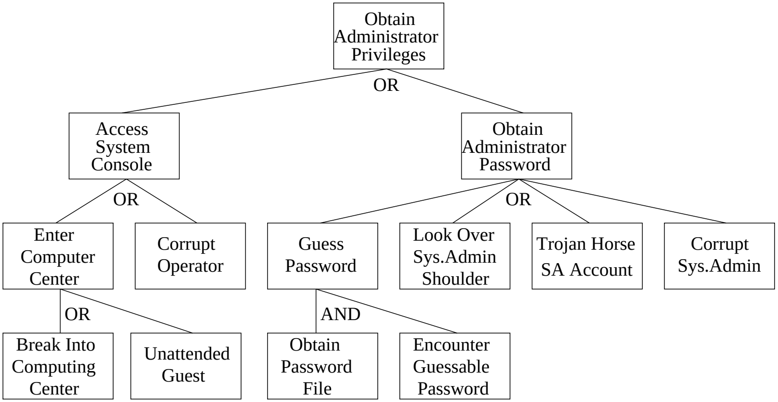
3rd October 2008

Outline

Attack Trees[1]



[1] J. D. Weiss 1991, Bruce Schneier 1999

Multi-parameter Attack Trees[2]

- Cost_i – the cost of the elementary attack, p_i – success probability
- π_i^- – the expected penalty in case the attack was unsuccessful
- π_i^+ – the expected penalty in case the attack was successful

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$$\mathrm{Outcome}_i = p_i \cdot \mathrm{Gains} - \mathrm{Cost}_i - p_i \cdot \pi_i^+ - (1 - p_i) \cdot \pi_i^-$$

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$$\begin{aligned} \mathrm{Costs} &= \mathrm{Costs}_1 + \mathrm{Costs}_2, \quad p = p_1 \cdot p_2, \quad \pi^+ = \pi_1^+ + \pi_2^+, \\ \pi^- &= \frac{p_1(1-p_2)(\pi_1^+ + \pi_2^-) + (1-p_1)p_2(\pi_1^- + \pi_2^+)}{1 - p_1 p_2} + \\ &+ \frac{(1-p_1)(1-p_2)(\pi_1^- + \pi_2^-)}{1 - p_1 p_2} \end{aligned}$$

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Foundations of Attack Trees[3]

1. Formal foundations of attack trees. Definition of attack components, attacks, attack suites, attack trees.
2. Transformations of attack trees. Transformations are proved to be sound and complete.

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    1. Conjunctive combinators calculate the value of attack from attack components.
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   3. When certain algebraic properties hold, we say that we have *distributive attributive domain*.
4. For example:
   1. $(\mathbb{N}, \min, +)$ could be interpreted as "cost of the cheapest attack".
   2. $(\mathbb{B}, \wedge, \vee)$ as "is the attack possible to complete".

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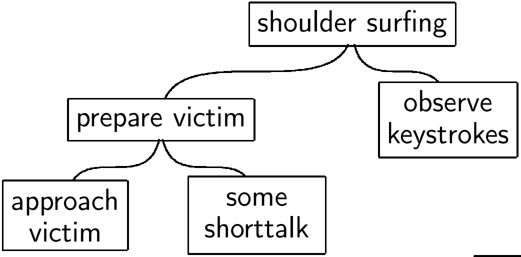
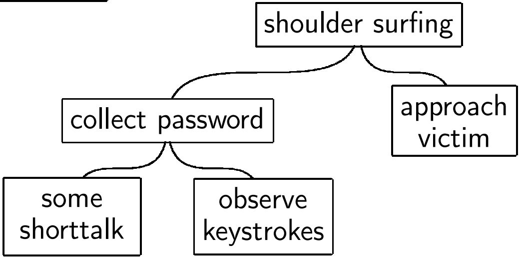
Equivalent attack trees





Equivalent attack trees

- $T_1 = A \vee (B \& C)$
- $T_2 = (A \vee B) \& (A \vee C)$
- $\text{Gains} = 10000$
- $p_A = 0.1,\ p_B = 0.5,\ p_C = 0.4$
- $\text{Expenses}_A = 1000,\ \text{Expenses}_B = 1500,\ \text{Expenses}_C = 1000$
- $\text{Outcome}_{\mathrm{T_1}} = 8000$
- $\text{Outcome}_{\mathrm{T_2}} = 6100$

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$$p_\sigma = \sum_{\substack{\rho \subseteq \sigma \\ \mathcal{F}(\rho := \mathsf{true}) = \mathsf{true}}} \prod_{X_i \in \rho} p_i \prod_{X_j \in \sigma \setminus \rho} (1 - p_j)\,. \tag{3}$$

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- $\text{Outcome}_{\sigma_3} = \text{Outcome}_T = 4380$

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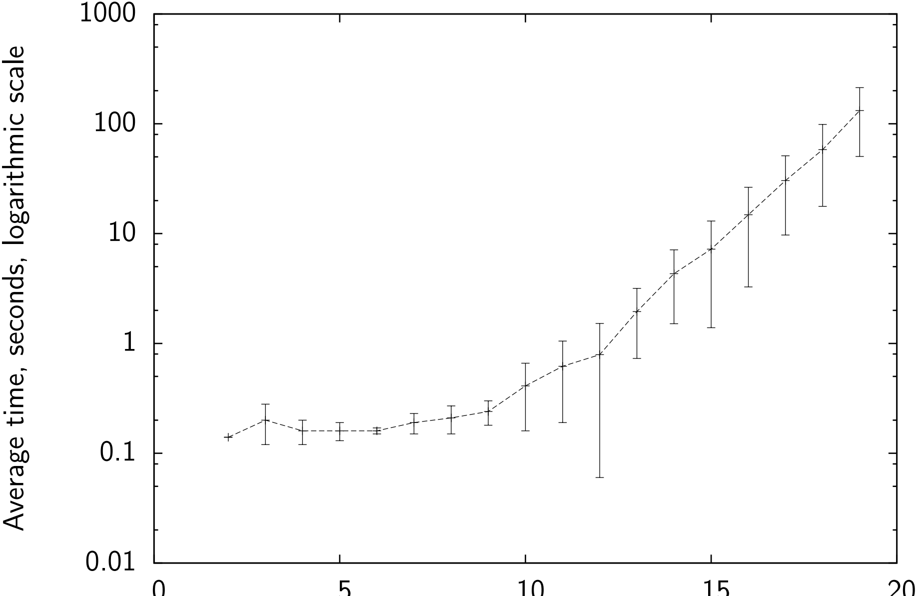
5. Possibly, parallelizing the calculations to multiple processors and multiple machines.


1000
100
10
1
0.1
0.01
Average time, seconds, logarithmic scale
0
5
10
15
20

How good is it?

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4. This raises the question about differences of propagating and non-propagating computations. Perhaps the Mauw and Oostdjik framework could be extended to non-propagating computations as well.

Conclusions

1. We presented new attack tree computation rules, which model attacker choices more precisely and provides bigger outcomes than the old model.
2. It is very difficult to calculate outcome of bigger trees. In some sense, this is the perfect solution for attack tree outcome calculation and we need to search for practical approximations now.
3. There are interesting questions about consistency with Mauw and Oostdijk framework model.