Non-locality and quantum games

Dmitry Kravchenko
University of Latvia

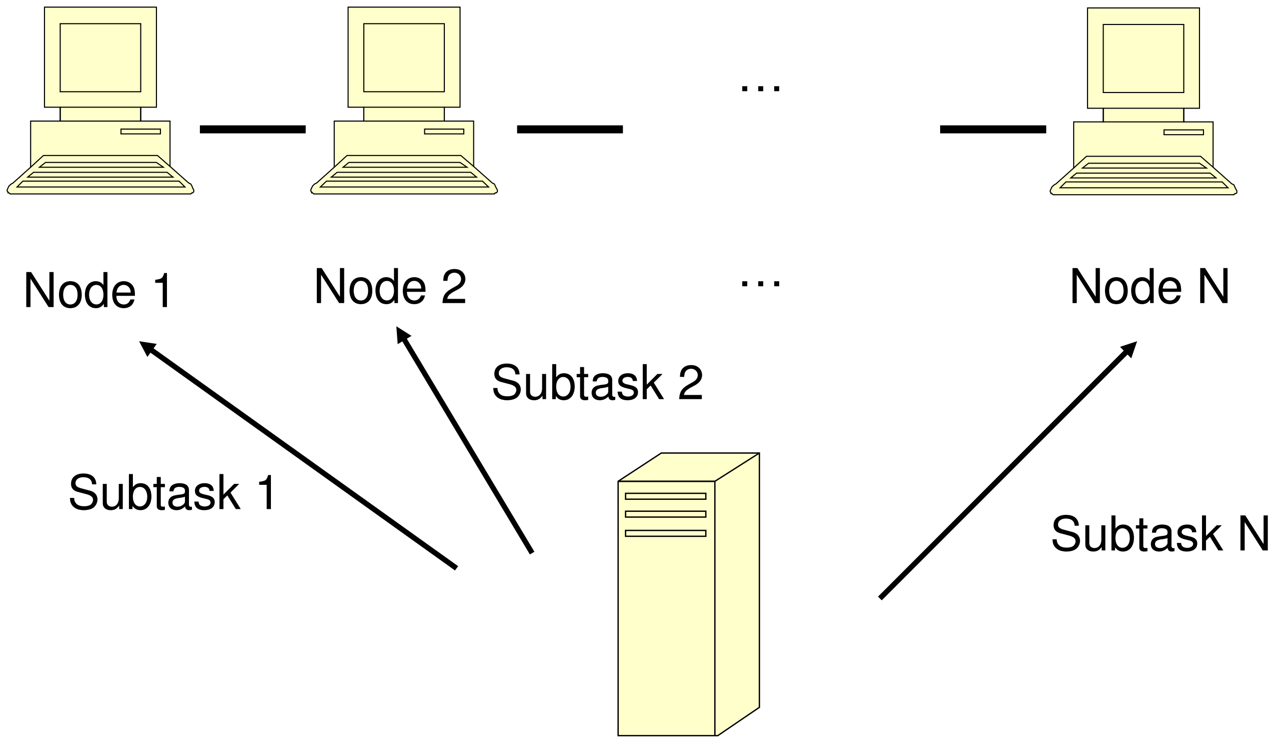
Theory days at Jõulumäe, 2008

Agenda

- Distributed computation
- Quantum mechanics basics
- Quantum non-local games
- Results and ideas

Distributed computation

- Main server distributes subtasks



Distributed computation

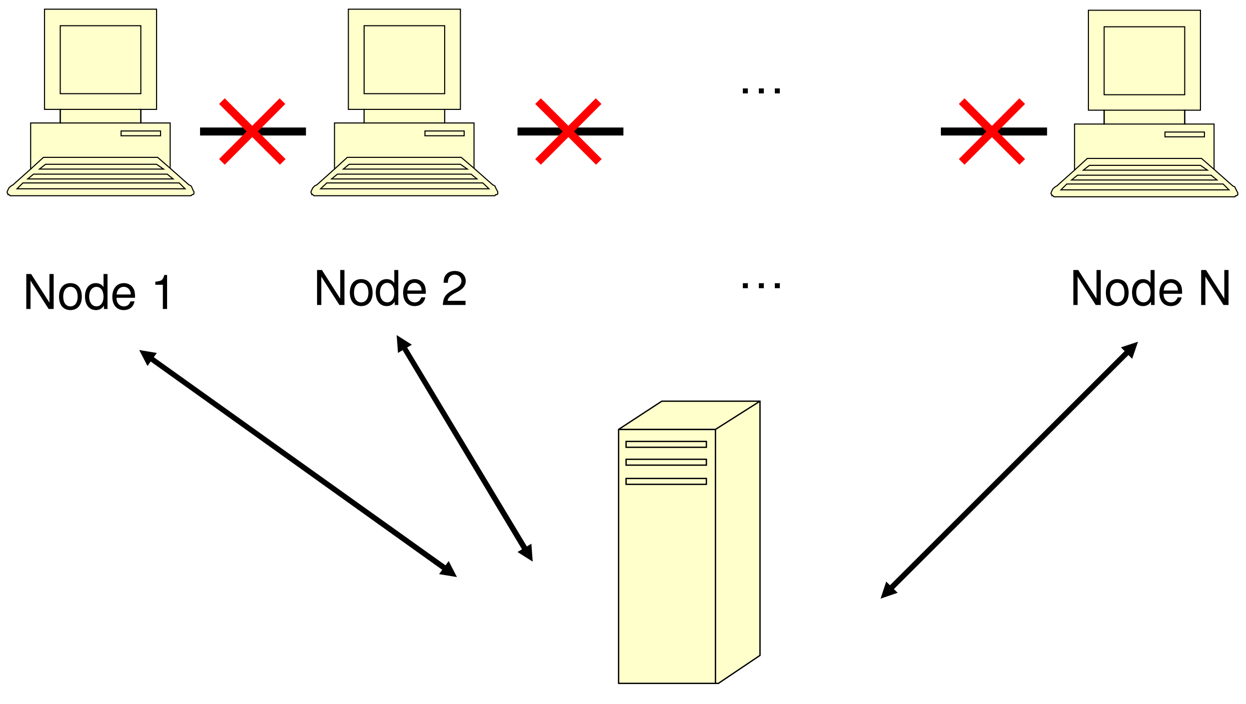
- If subtasks are correlated but communications between nodes is allowed nodes can compute any computable function
- However node abilities to communicate can be limited or even completely prohibited

Distributed computation

- Long distances between nodes (space)
- Environment (ocean)
- Communication takes too much energy
- Etc.

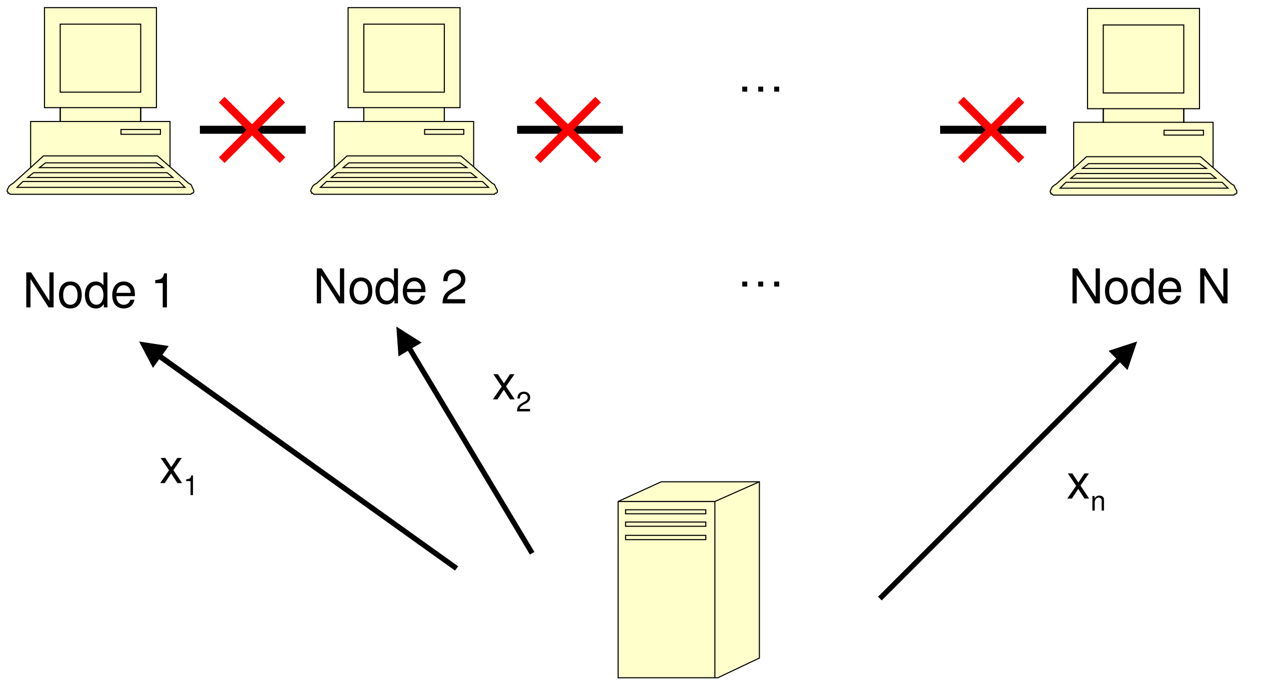
Distributed computation

- If communications are prohibited some distributed functions can not be computed (with certainty)



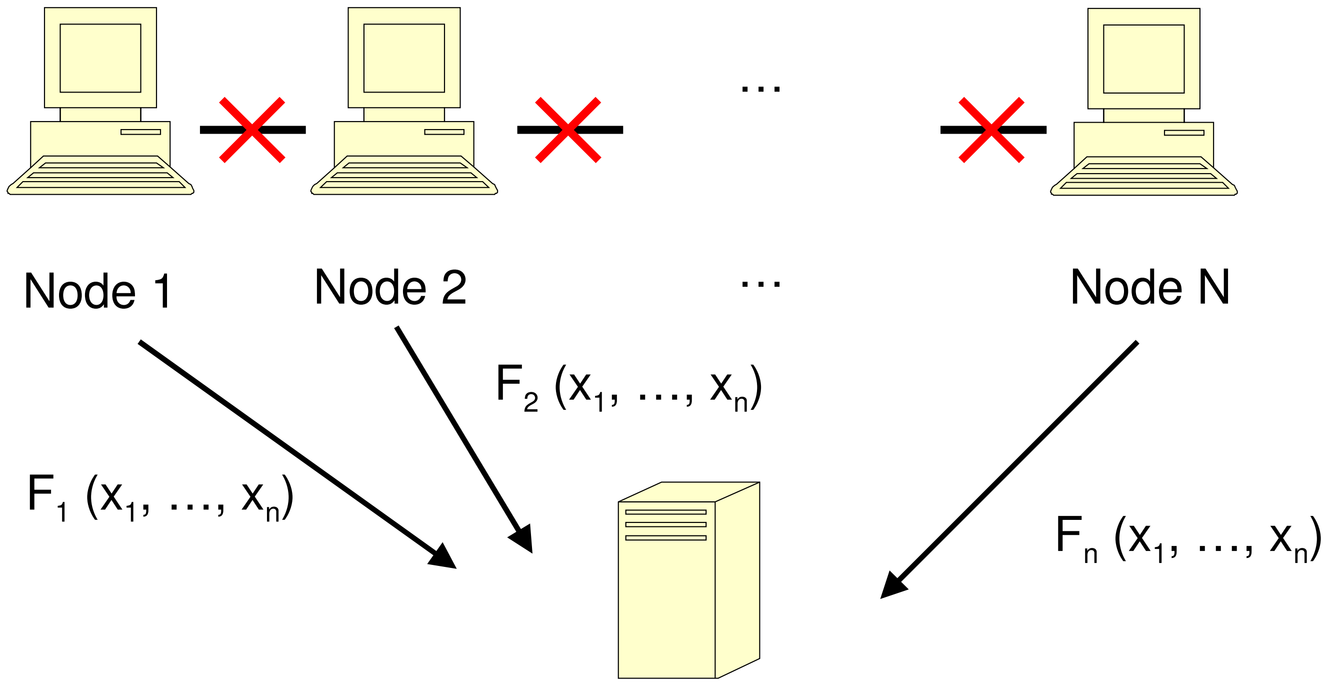
Computational model

- More formally: nodes compute a set of functions on shared data



Computational model

- More formally: nodes compute a set of functions on shared data



Classical and quantum bits

- Classical bit
  - Regardless a physical representation can be 0 or 1
- Quantum bit
  - Regardless a physical representation can be 0, 1 or a superposition of both

Quantum bits : superposition

- If a quantum bit can be in state 0 and 1

  it can also be in state $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$

  where α and β are complex numbers called probability amplitudes.

Quantum bits : measurement

- If a quantum bit can be in state 0 and 1

  it can also be in state $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$

- Measuring the bit the probability of outcome 0 is $|\alpha|^2$ and the probability of outcome 1 is $|\beta|^2$.

- α and β must be constrained by the equation

$$|\alpha|^2 + |\beta|^2 = 1$$

Quantum bits : generalization

- If a quantum system can be in states $|\varphi_1\rangle, \ldots, |\varphi_n\rangle$

  it can also be in state $|\varphi\rangle = \sum_{i=1}^{n} \alpha_i |\varphi_i\rangle$

- Measuring the system the probability of outcome i is $|\alpha_i|^2$

- α_i must be constrained by the equation

$$\sum_{i=1}^{n} |\alpha_i|^2 = 1$$

Entanglement

- Entanglement is a non-local property that allows a set of qubits to express higher correlation than is possible in classical systems.

- It gives rise to some of the most counterintuitive phenomena of quantum mechanics

Entanglement

- We have a system consisting of two bits.
- In classical case it is always possible to describe a state of each bit.
- In quantum case the system can be in a state there individual qubits do not have their own state.

Entanglement : example

- We have a system consisting of two qubits.
- The system can be in any superposition

$$|\varphi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

- For example, in superposition

$$|\varphi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Entanglement

- We have a pair of entangled qubits

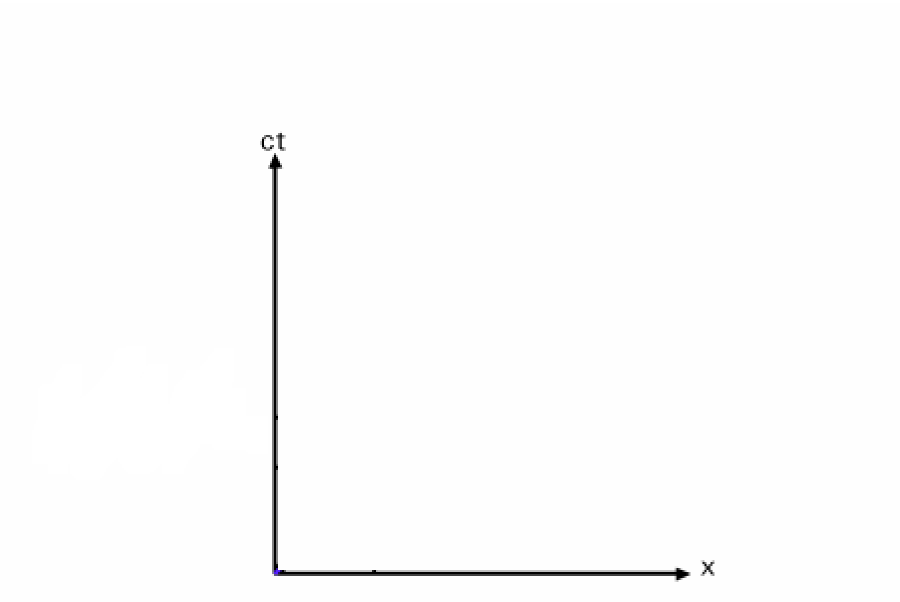
$$|\varphi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- As qubits are particles they can be physically separated

- If we measure one of qubits other will “get” same state

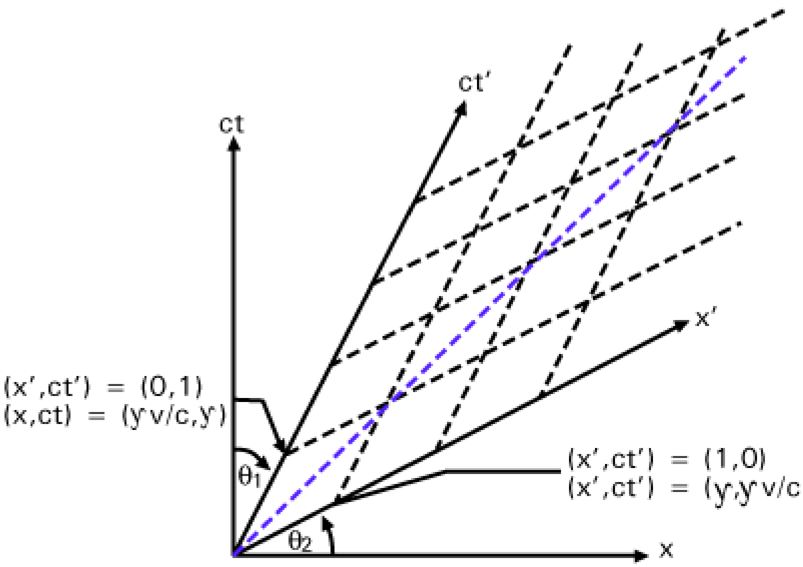
Synchronism

- We can not transmit information using entanglement as this violates relativity theory
- Static observer has horizontal worldline



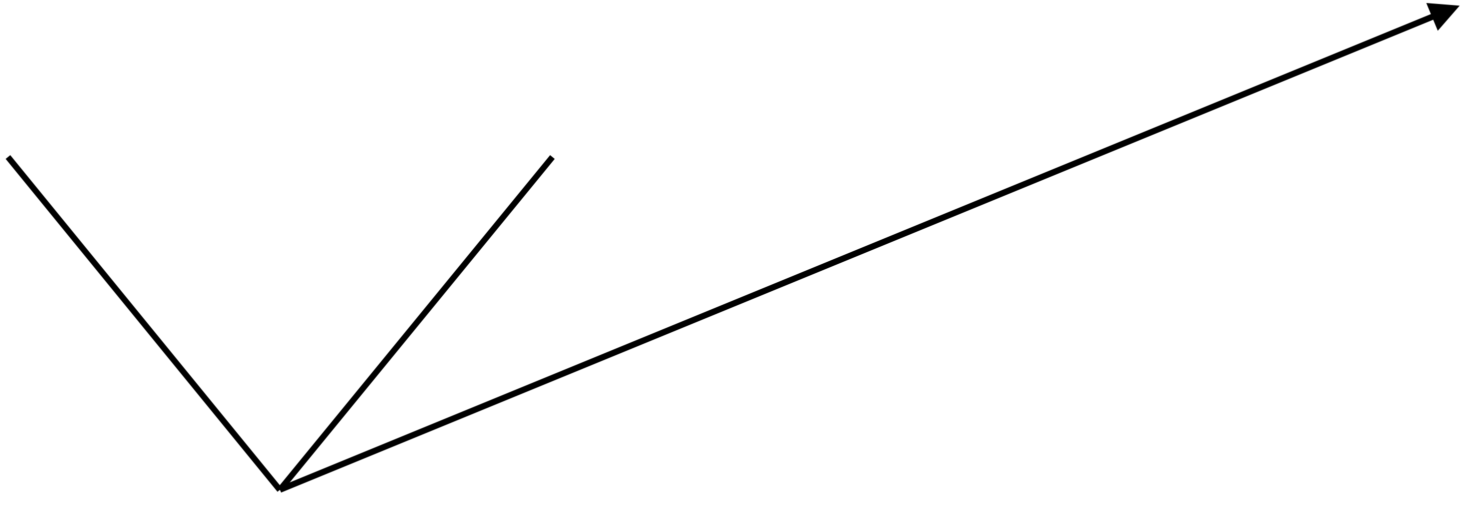
Synchronism

- We can not transmit information using entanglement as this violates relativity theory
- Static observer has horizontal worldline
- Observer in motion has inclined worldline



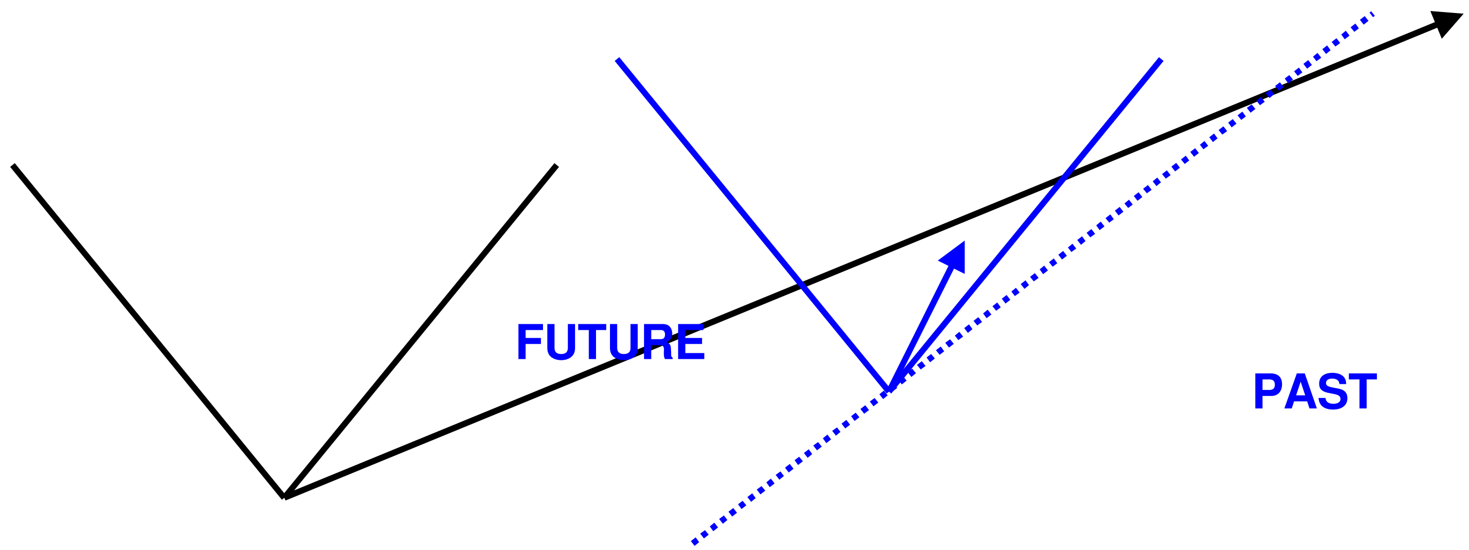
Synchronism

- If one exceeds speed of light, there may exist an observer that has opposite time flow
- Cause-and-effect law is violated



Synchronism

- If one exceeds speed of light, there may exist an observer that has opposite time flow
- Cause-and-effect law is violated



Entanglement once again

- We have a pair of entangled qubits

$$|\varphi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- As qubits are particles they can be physically separated

- If we measure one of qubits other will “get” same state

Entanglement once again

- We have a pair of entangled qubits

$$|\varphi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- As qubits are particles they can be physically separated

- If we measure one of qubits other will “get” same state

IMMEDIATELY !

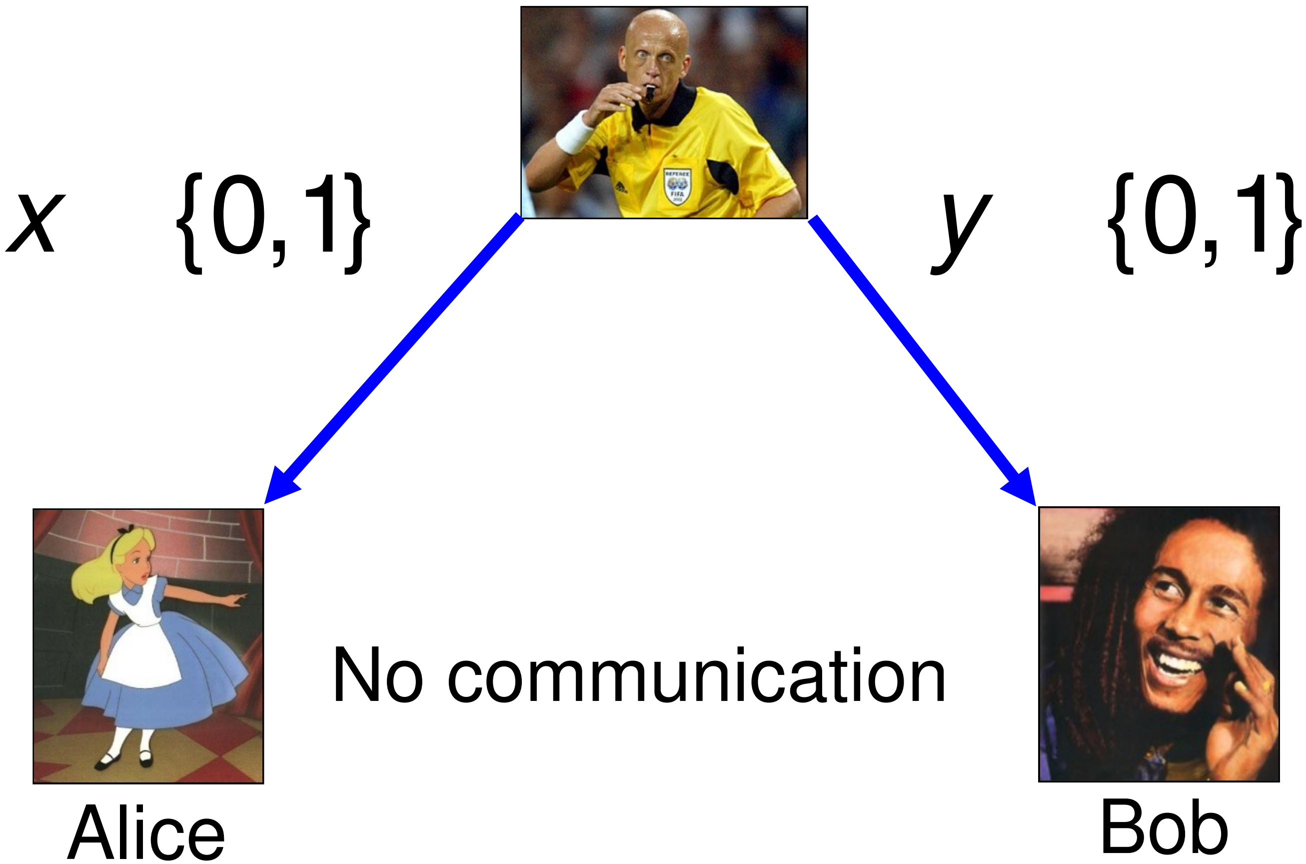
The CHSH game

John **C**lauser,
Michael **H**orne,
Abner **S**himony and
Richard **H**olt

1969

CHSH inequality

The CHSH game



The CHSH game

$a \quad \{0,1\}$ $b \quad \{0,1\}$

Alice

No communication

Bob

$x*y = a \oplus b$

The CHSH game

$a \quad \{0,1\}$ $b \quad \{0,1\}$

Alice

No communication

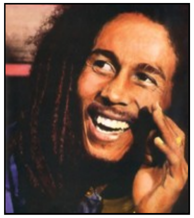
Bob

|Input|=2 ⇔ |Output|=1

Best classical strategy

Input		
x*y	0	1
0	0	0
1	0	1

Output		
a⊕b		

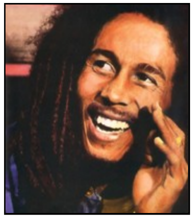


Best classical strategy

Input

x*y	0	1
0	0	0
1	0	1

Output



a⊕b	0	0
0	0	0
0	0	0

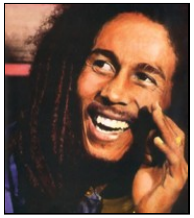
Best classical strategy

Input

x*y	0	1
0	**0**	**0**
1	**0**	1

Output

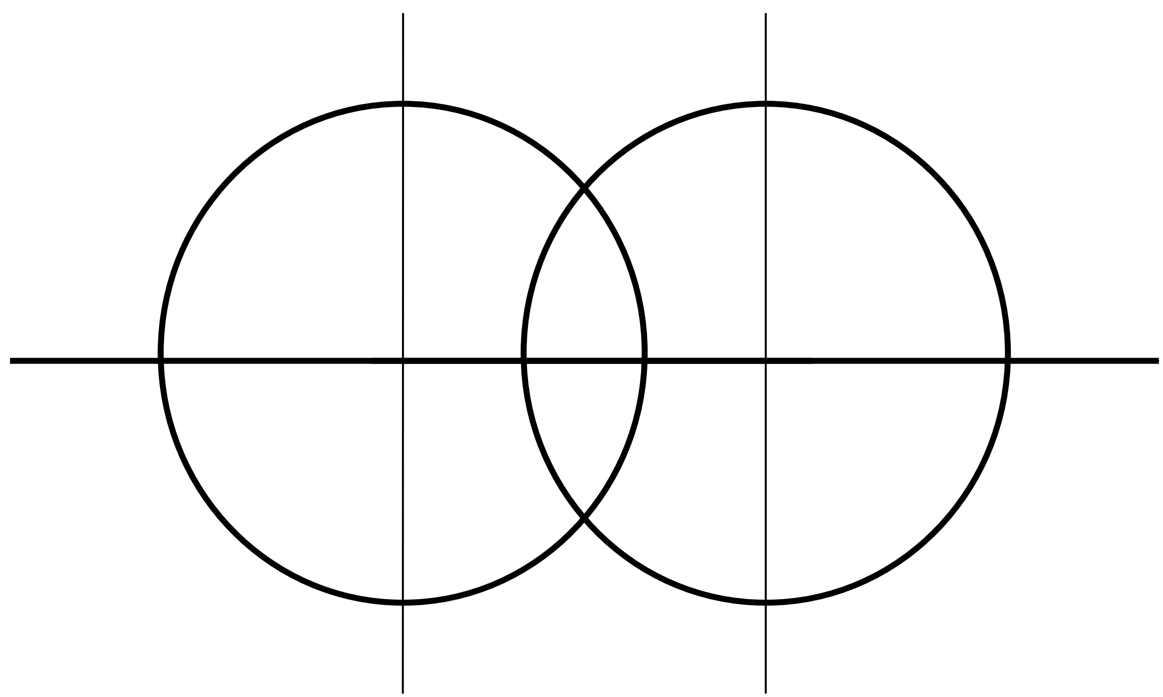
a⊕b	0	0
0	**0**	**0**
0	**0**	0



Best classical strategy

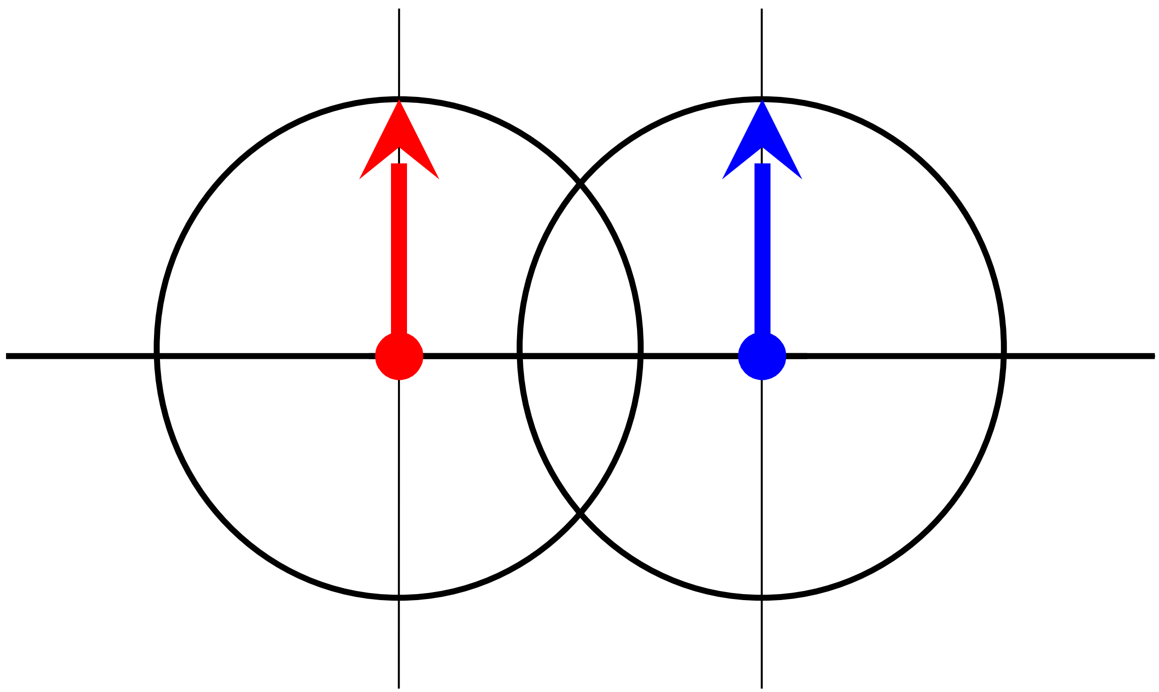
$$\Pr[\text{Alice \& Bob win}] = 3/4$$

Entanglement : measurement



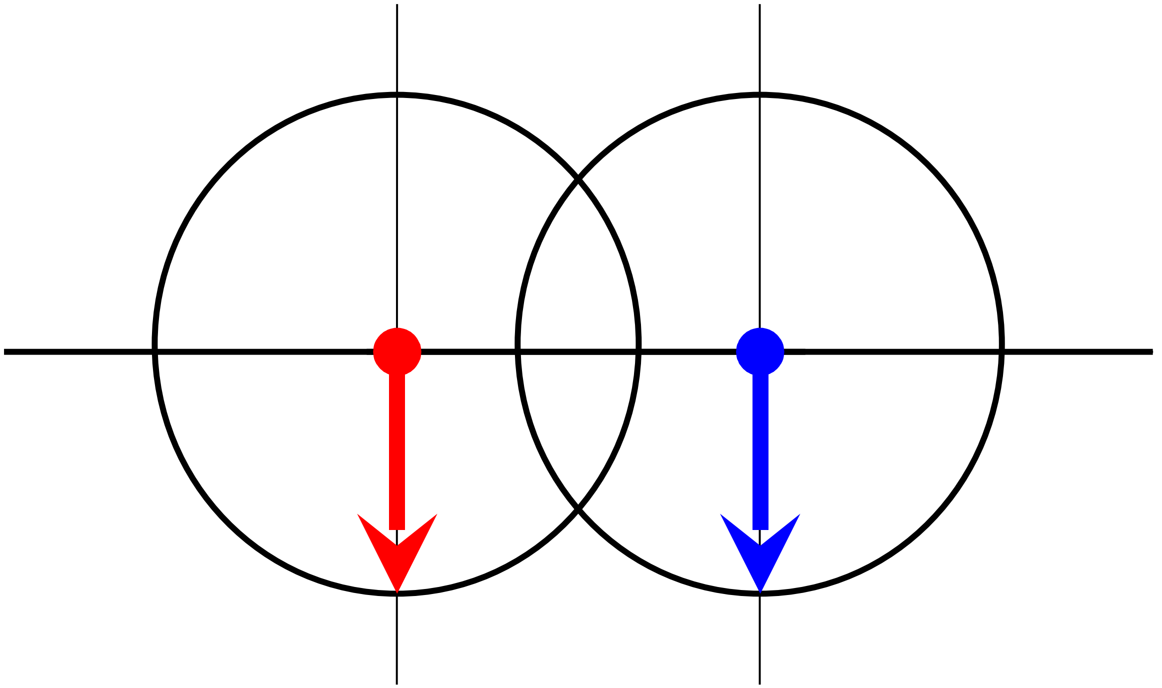
Entanglement : measurement

Either ↑↑



Entanglement : measurement

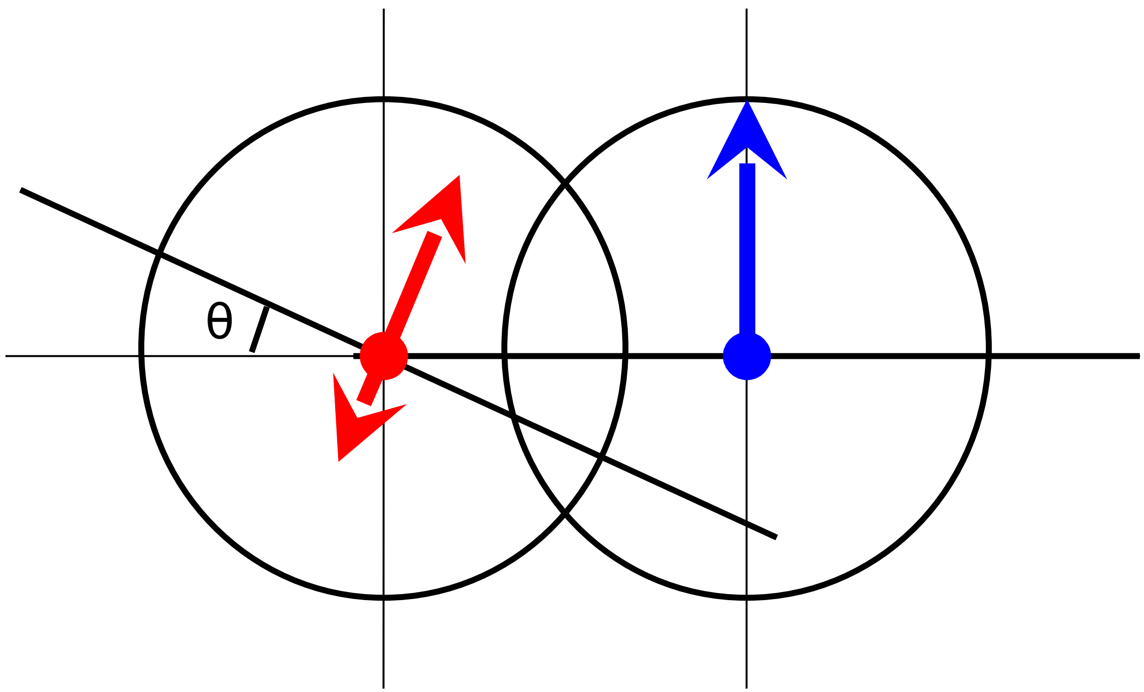
Either ↑↑ or ↓↓



Entanglement : measurement

↑↑ ($\cos^2\theta$)

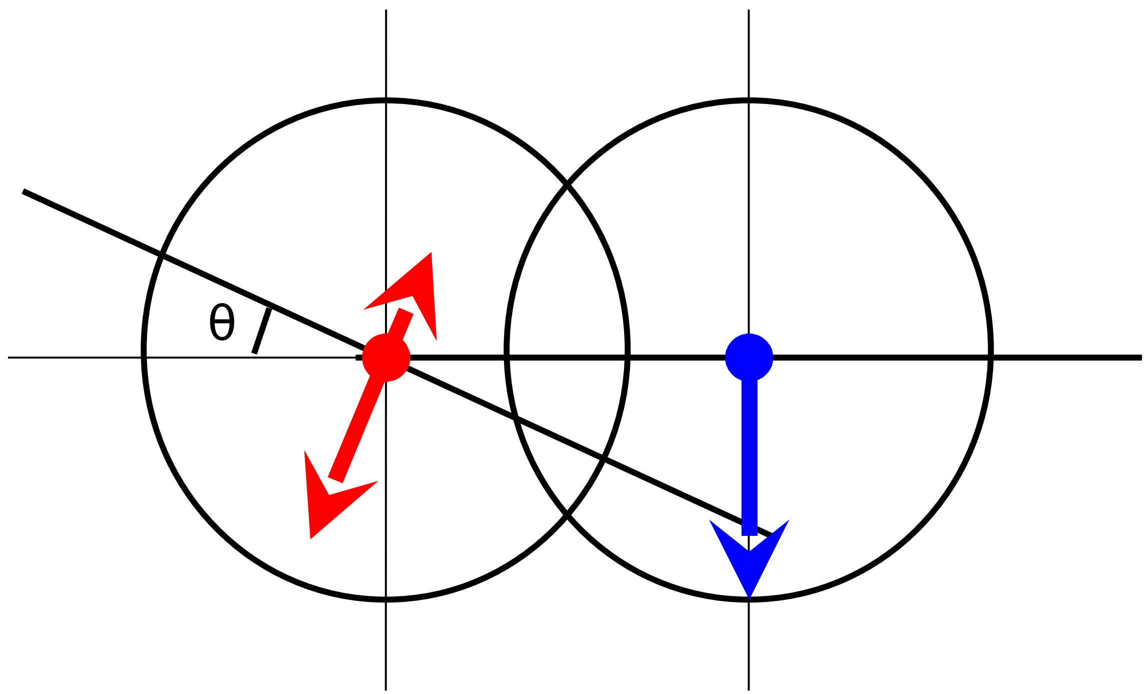
↓↑ ($\sin^2\theta$)



Entanglement : measurement

$\uparrow\uparrow$ $(\cos^2\theta)$ $\downarrow\downarrow$ $(\cos^2\theta)$

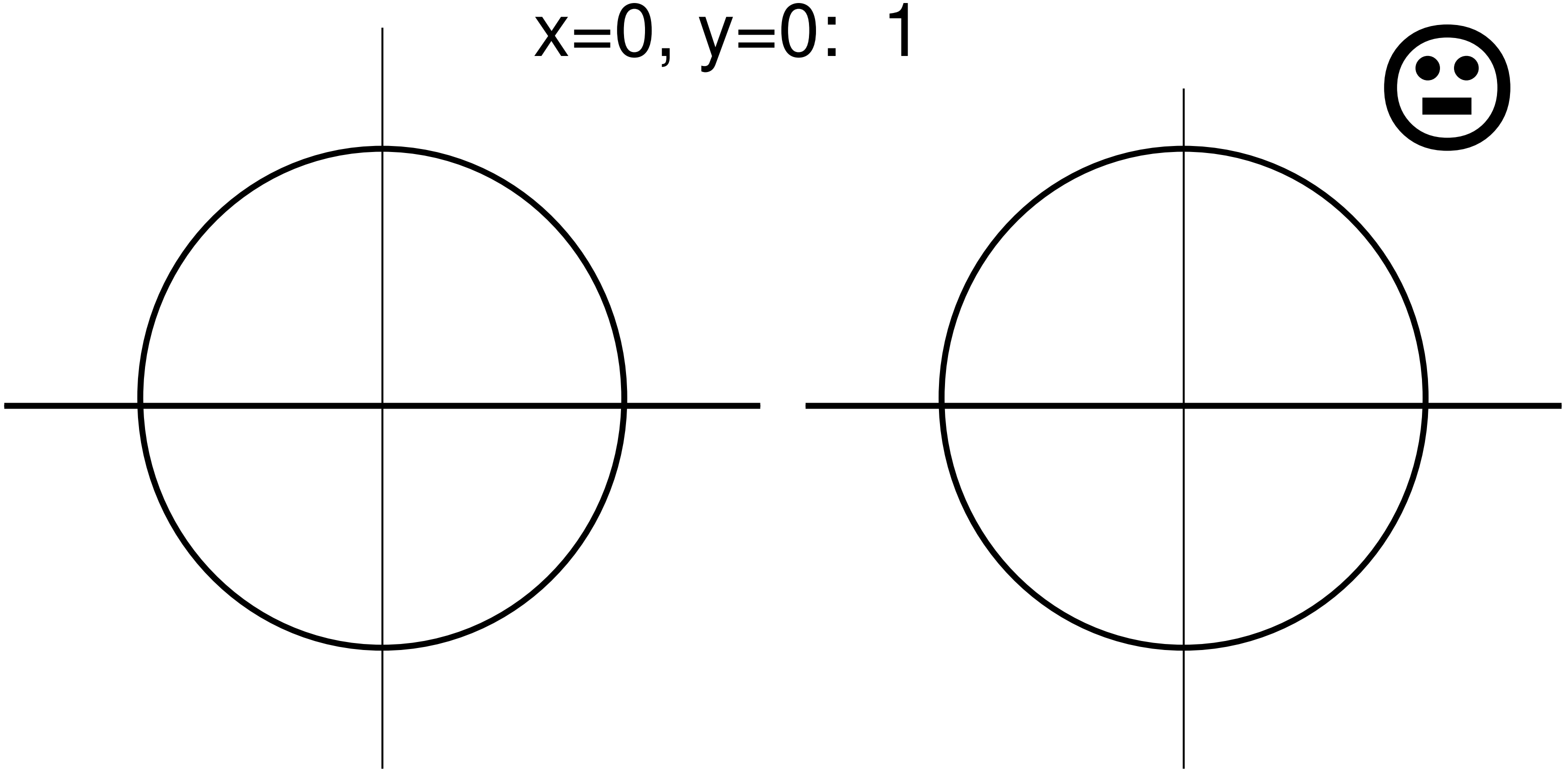
$\downarrow\uparrow$ $(\sin^2\theta)$ $\uparrow\downarrow$ $(\sin^2\theta)$



Quantum strategy

- What if Alice and Bob share 2 qubit system?

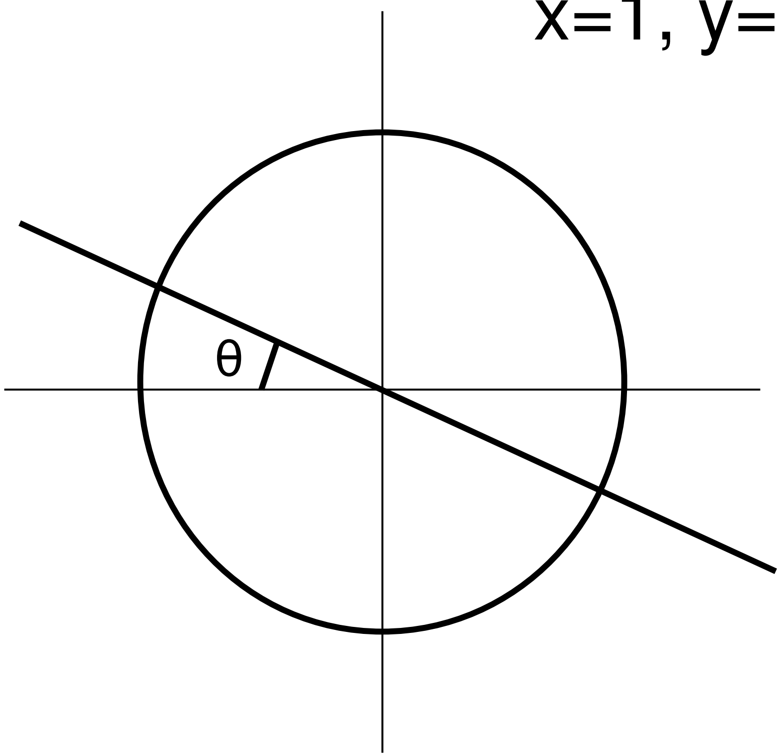
x=0, y=0: 1

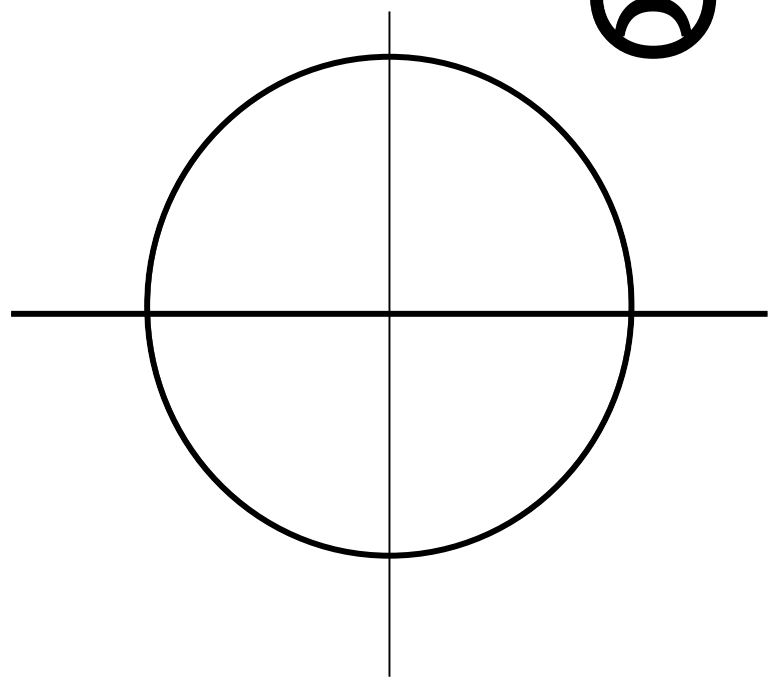


Quantum strategy

- What if Alice and Bob share 2 qubit system?

$x=1, y=0$: $1 - \sin^2 \theta$ ☹

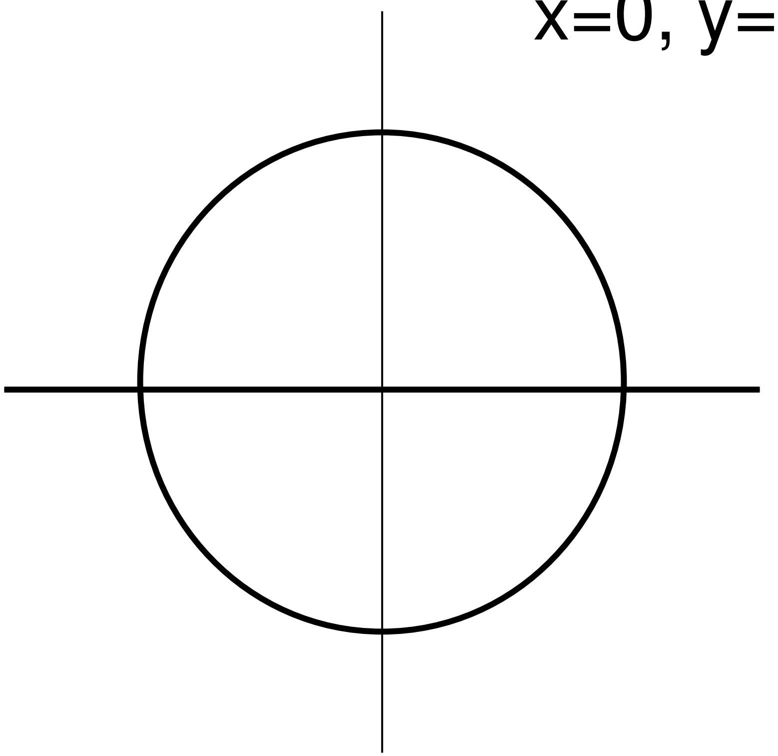


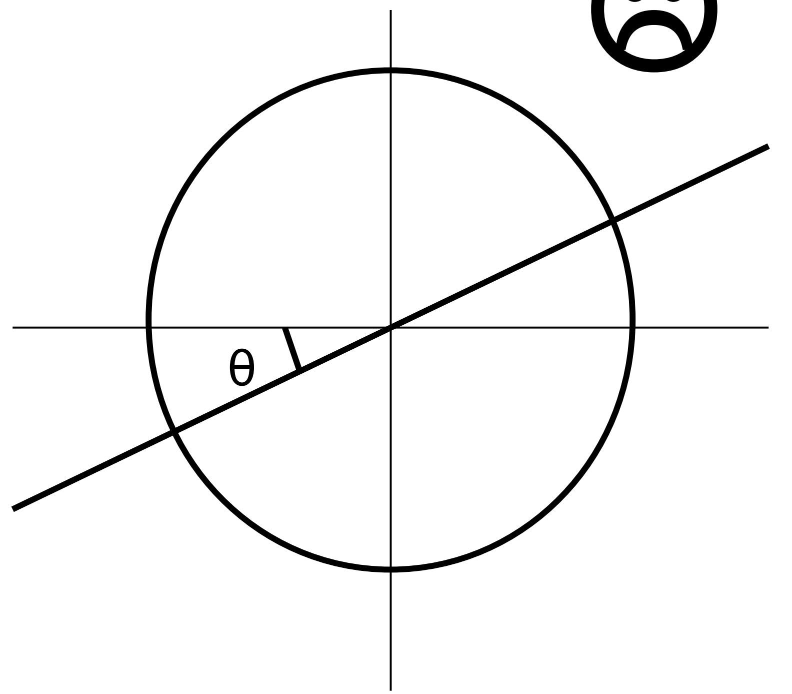


Quantum strategy

- What if Alice and Bob share 2 qubit system?

$x=0$, $y=1$: $1 - \sin^2 \theta$ ☹

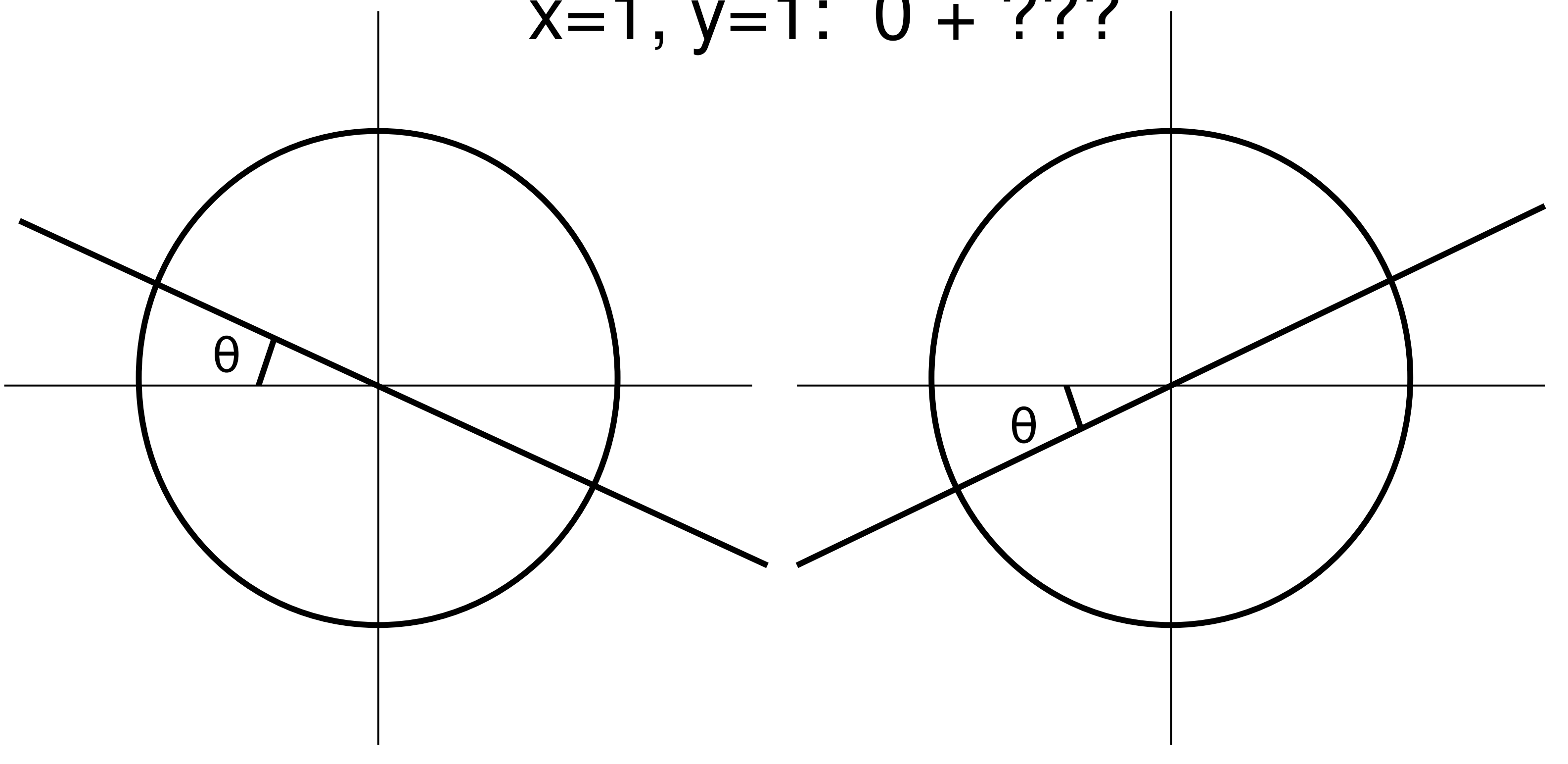




Quantum strategy

- What if Alice and Bob share 2 qubit system?

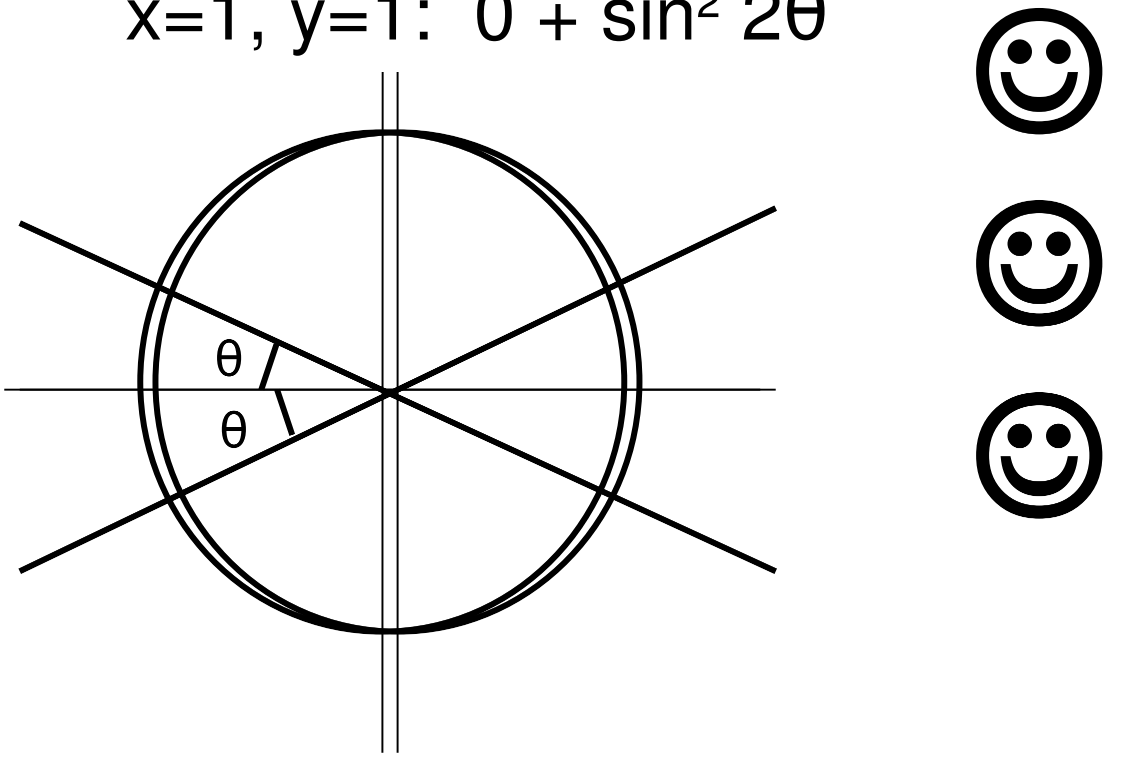
x=1, y=1: 0 + ???



Quantum strategy

- What if Alice and Bob share 2 qubit system?

$x=1, y=1: \; 0 + \sin^2 2\theta$



Best quantum strategy

😐 + ☹ + ☹ + ☺☺☺ = ☺

Pr[Alice & Bob win*]

$= \cos^2(\pi / 8) \quad 0.85$

* when they share $|\Phi^-$

The CHSH game

$$0.85 > 3/4$$

- There is a quantum strategy which is better than any classical strategy

- This is one of the cores of quantum non-locality

Games of four

- All symmetrical games can be written in form like

$$|\mathbf{Input}| \in \{\mathbf{0,1,4}\} \quad \Leftrightarrow \quad |\mathbf{Output}| \in \{\mathbf{0,2,3}\}$$

Games of four

- All symmetrical games can be written in form like

  |**Input**| ∈ {**0,1,4**} ⇔ |**Output**| ∈ {**0,2,3**}

- Best known "quantum achievements" for such 4 player games are:

{1} ⇔ {2}	0,75 vs 0,796875
{3} ⇔ {2}	0,75 vs 0,796875
{0,3} ⇔ {2}	0,6875 vs 0,734375
{1,4} ⇔ {2}	0,6875 vs 0,734375

Quantum non-local games

- Other examples of non-local games need research
- The main task: define function pairs set that represent games which allow non-local quantum tricks
- In particular, describe the [quite strict!] restriction that comes from relativity theory

Thank you!