Formalizing Attribute Grammars and Circularity Checking

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Previous work and motivation

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- Parsing (context-free languages)

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A: Attribute grammars

Q: How to be sure that semantical value can be found for all parse trees?
A: Well-definedness property

Example: Semantics of binary numbers

$$\mathtt{N} \to \mathtt{L}$$
$$\mathtt{L}_1 \to \mathtt{L}_2\mathtt{B}$$

$$\mathtt{L} \to \mathtt{B}$$
$$\mathtt{B} \to \mathtt{0}$$
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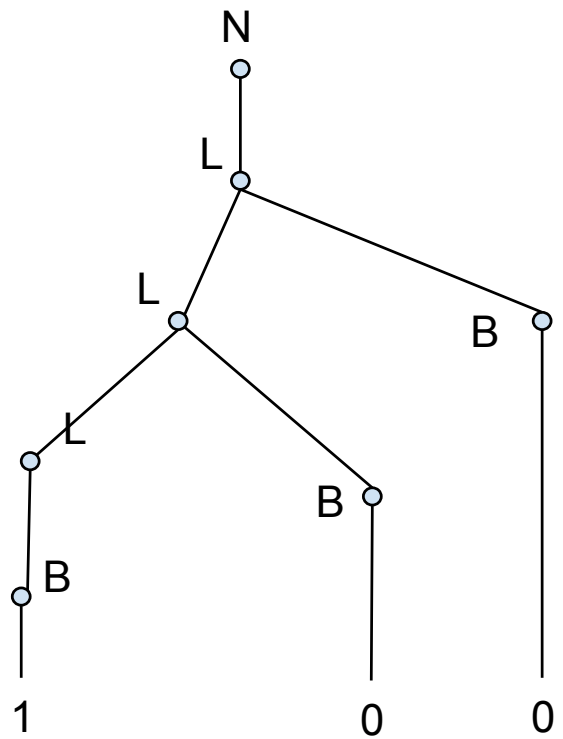

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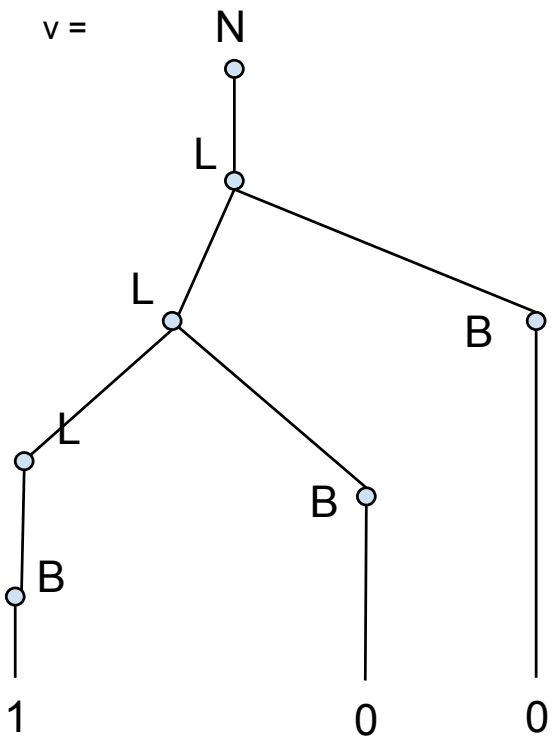

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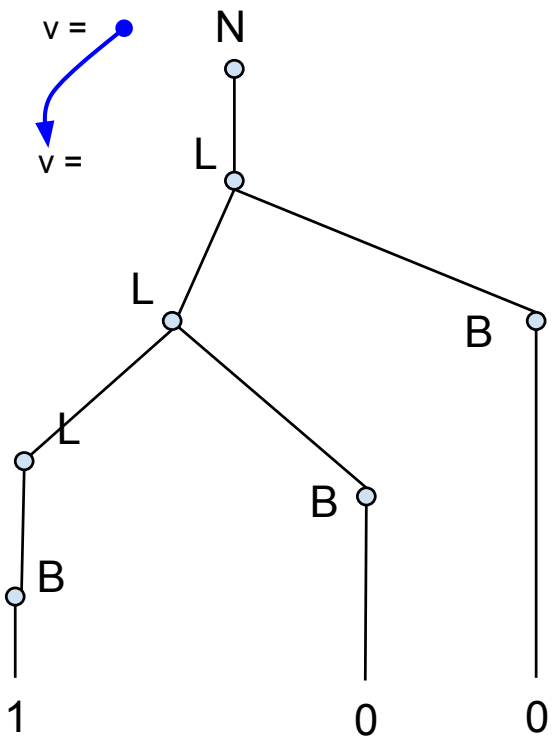

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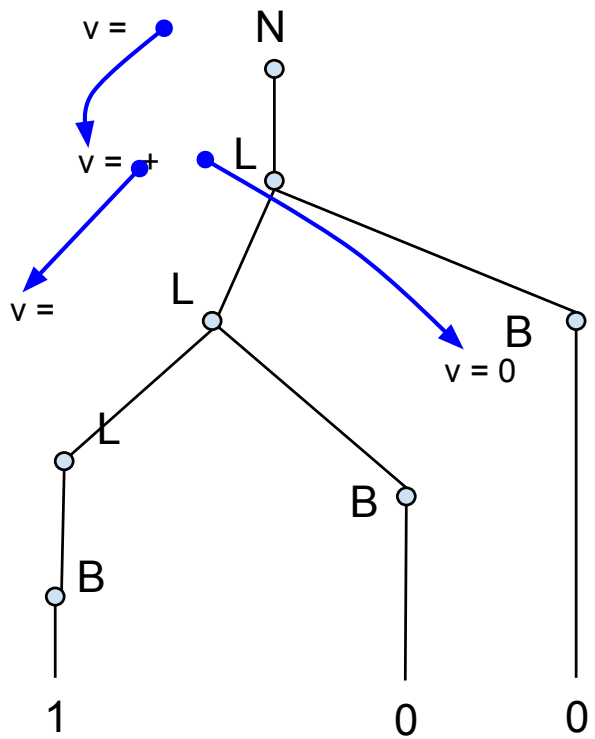

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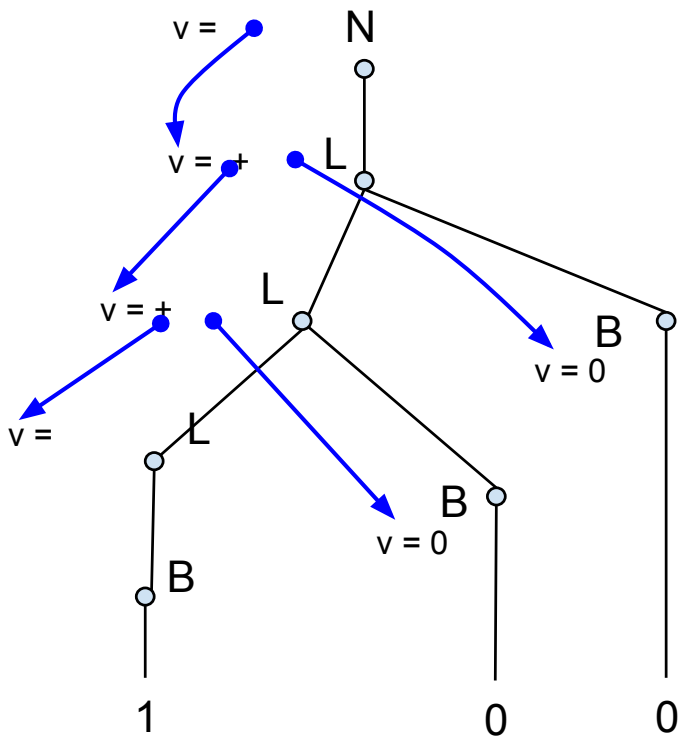

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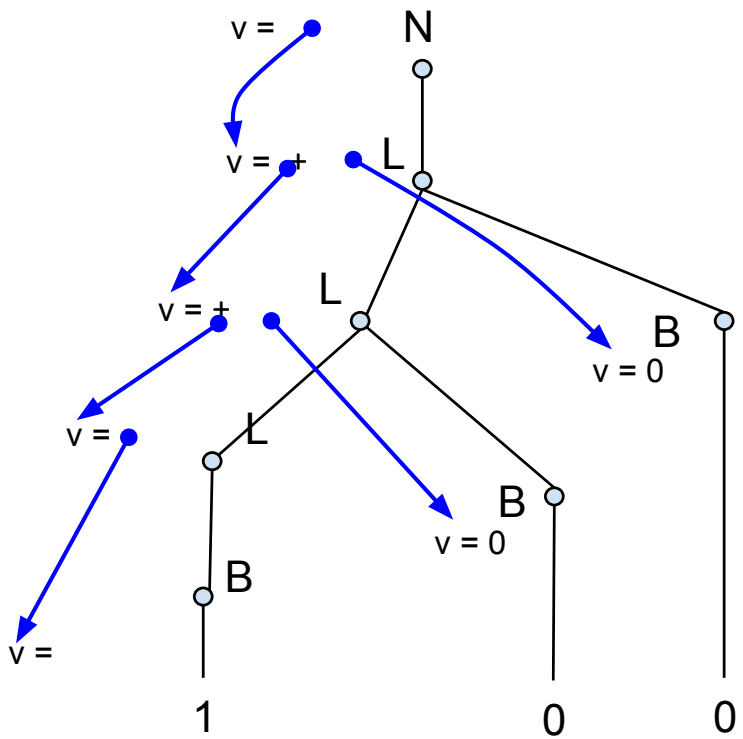

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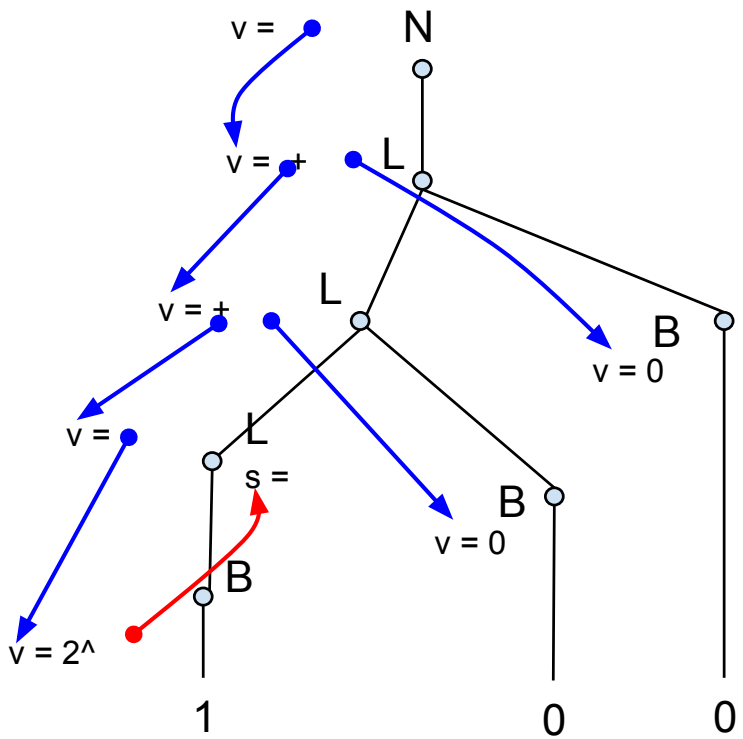

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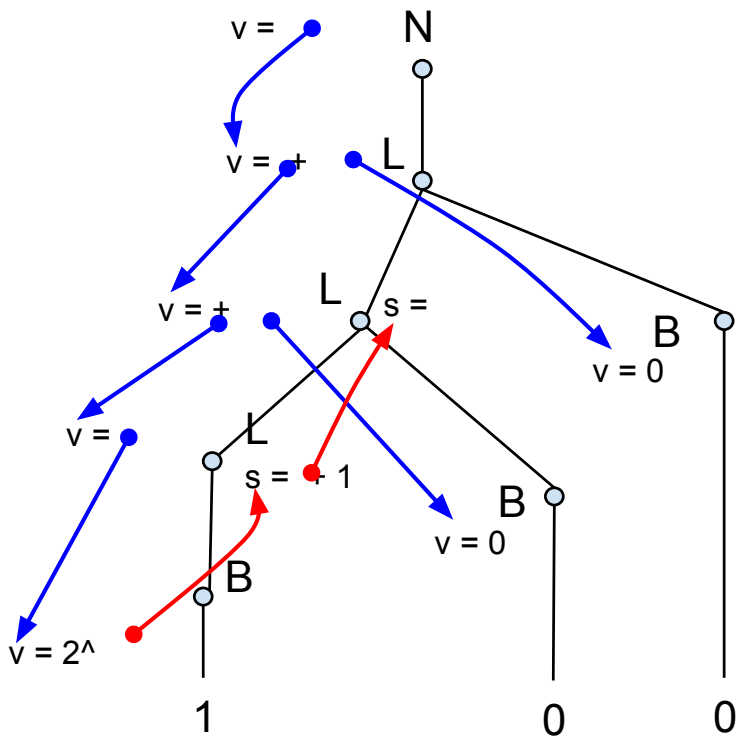

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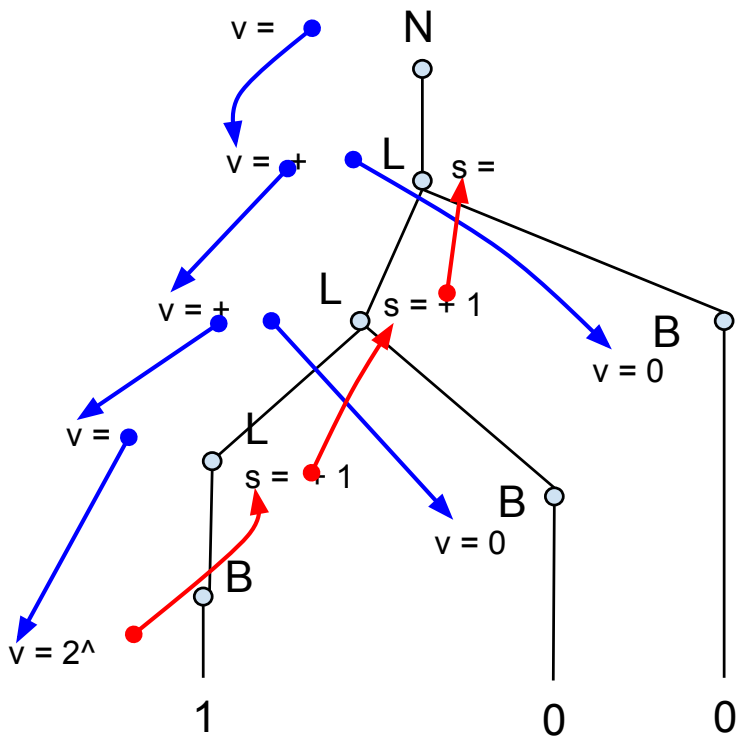

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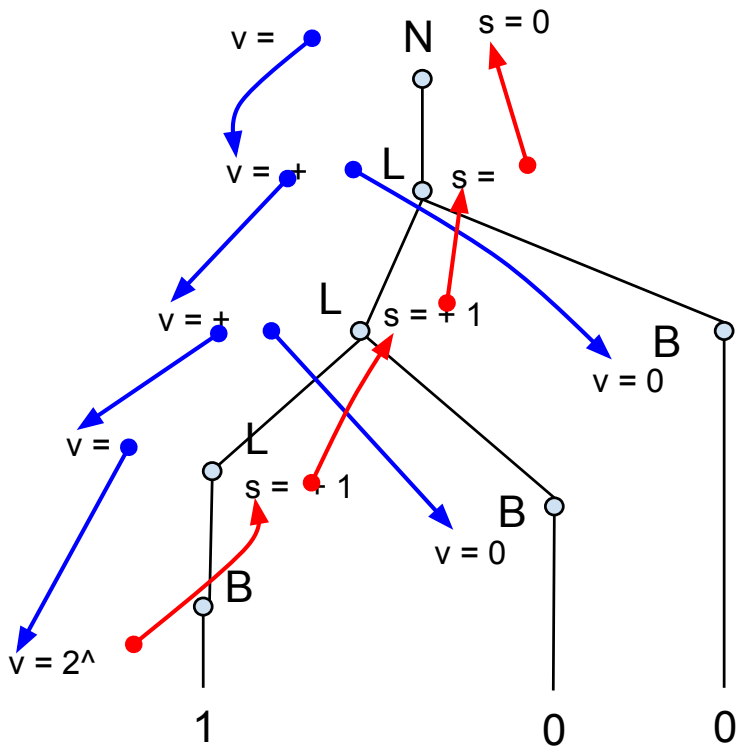

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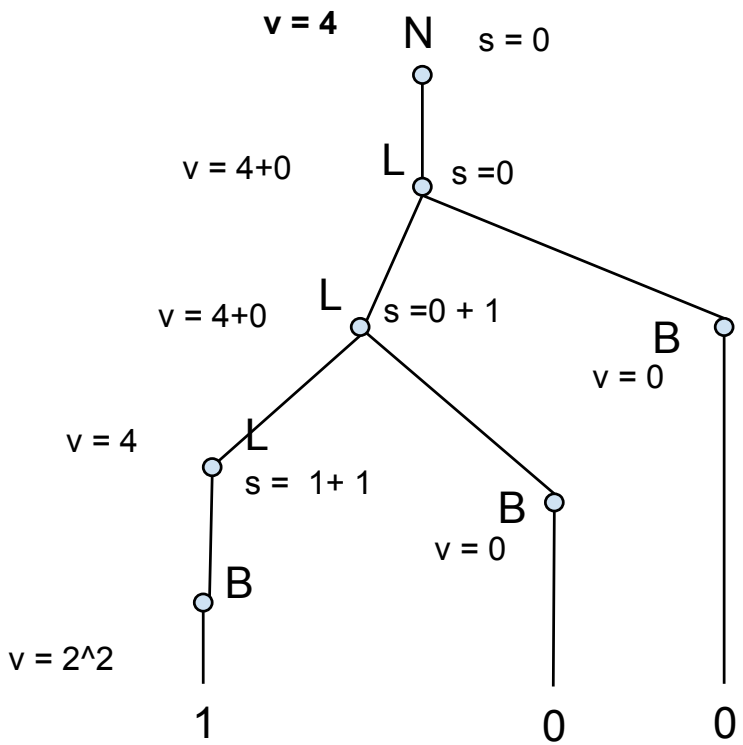

Basic definitions: Parse trees

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Attr : Set
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mutual
  Forest = List (∃(X : N), Tree X)

  data Tree : N → Set where
    node : ∀ {X} → (f : Forest)
         → Rule X (map proj₁ f) → Tree X
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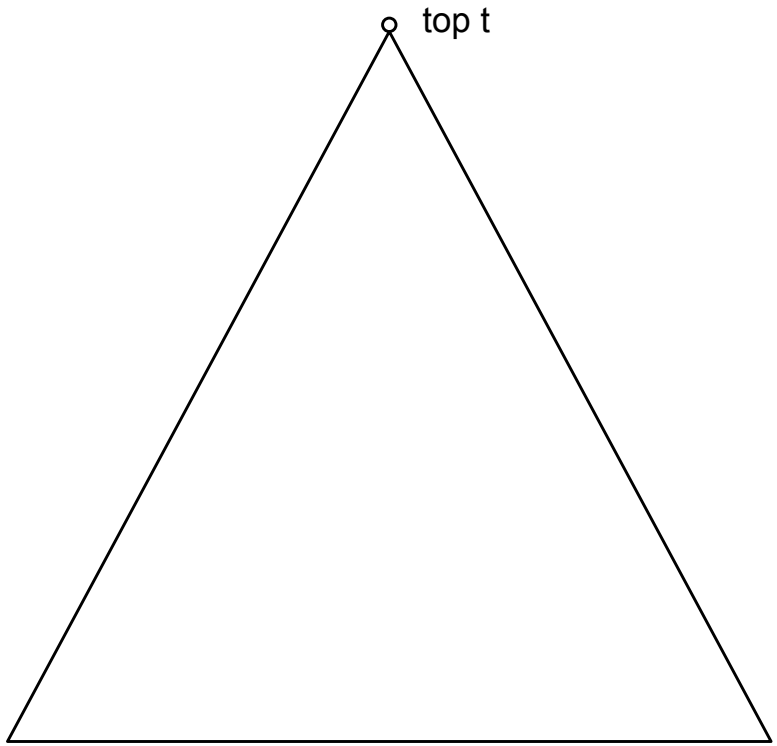

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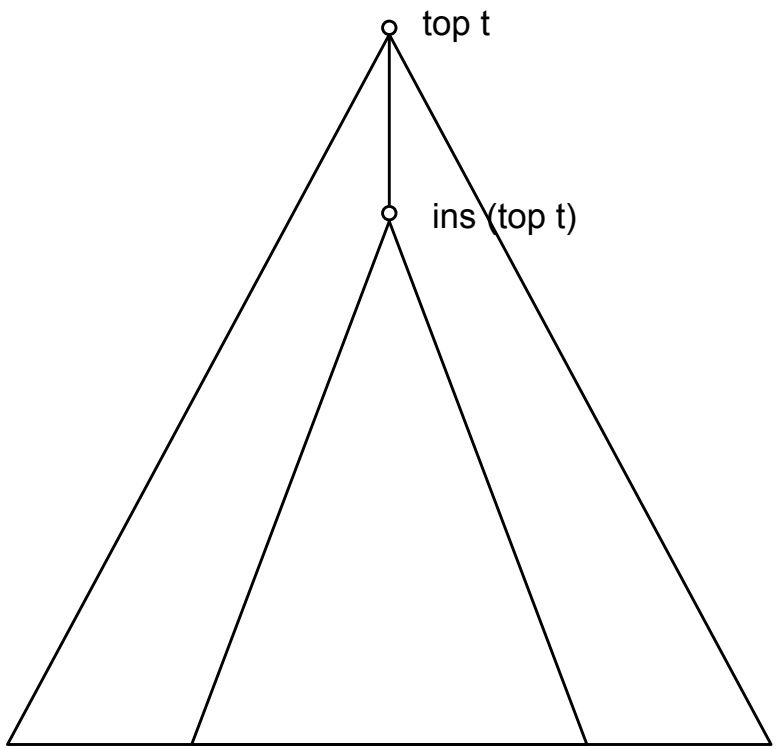

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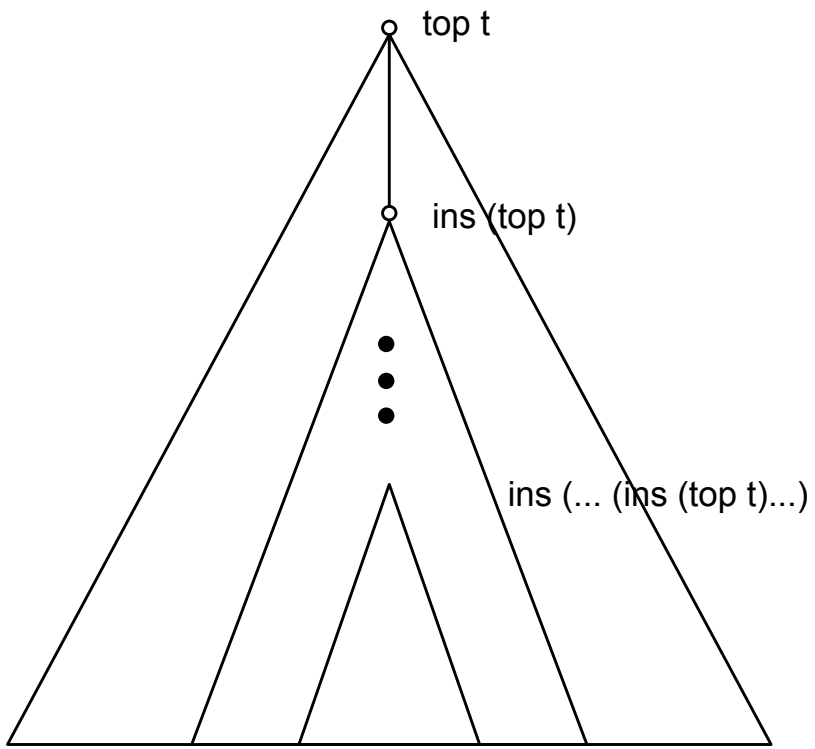

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b|[ p , a₁ ] ⇓ [ ins p , a₂ ]
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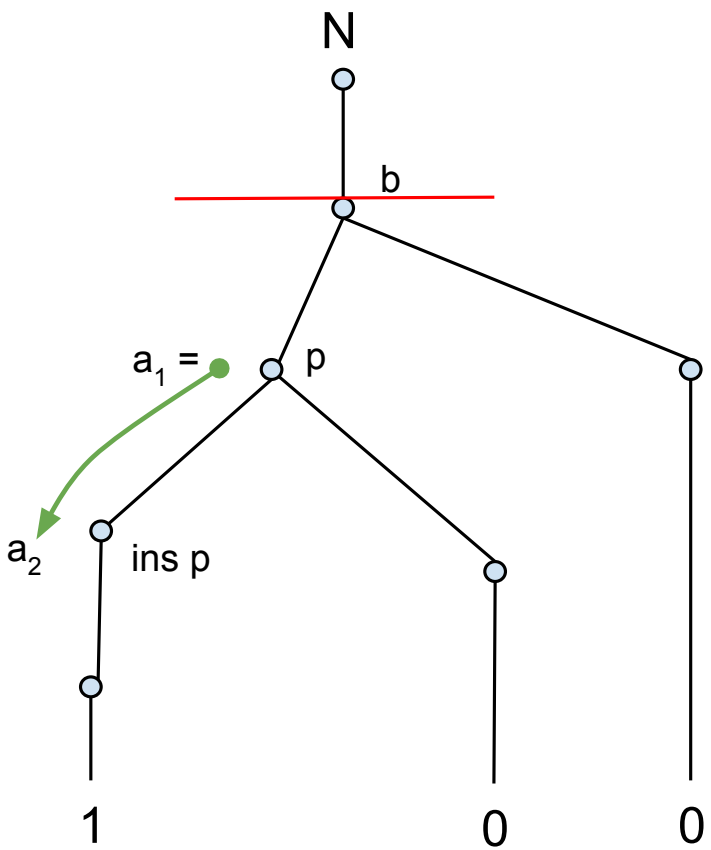

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```
q |[ q , v ] ⁺↝ [ q , s ]
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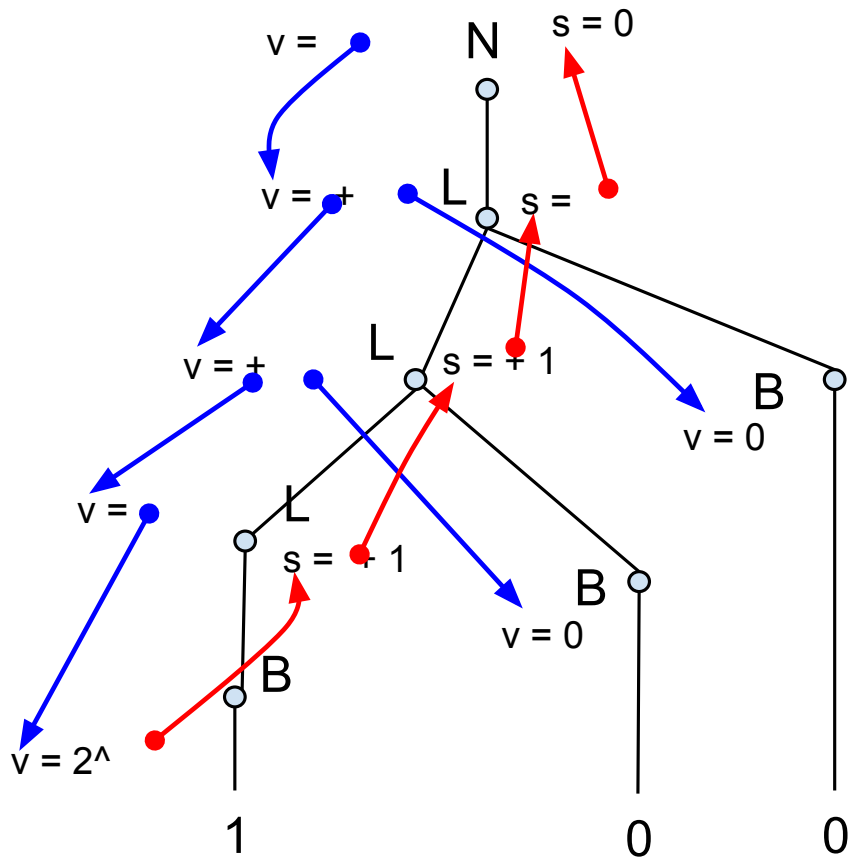

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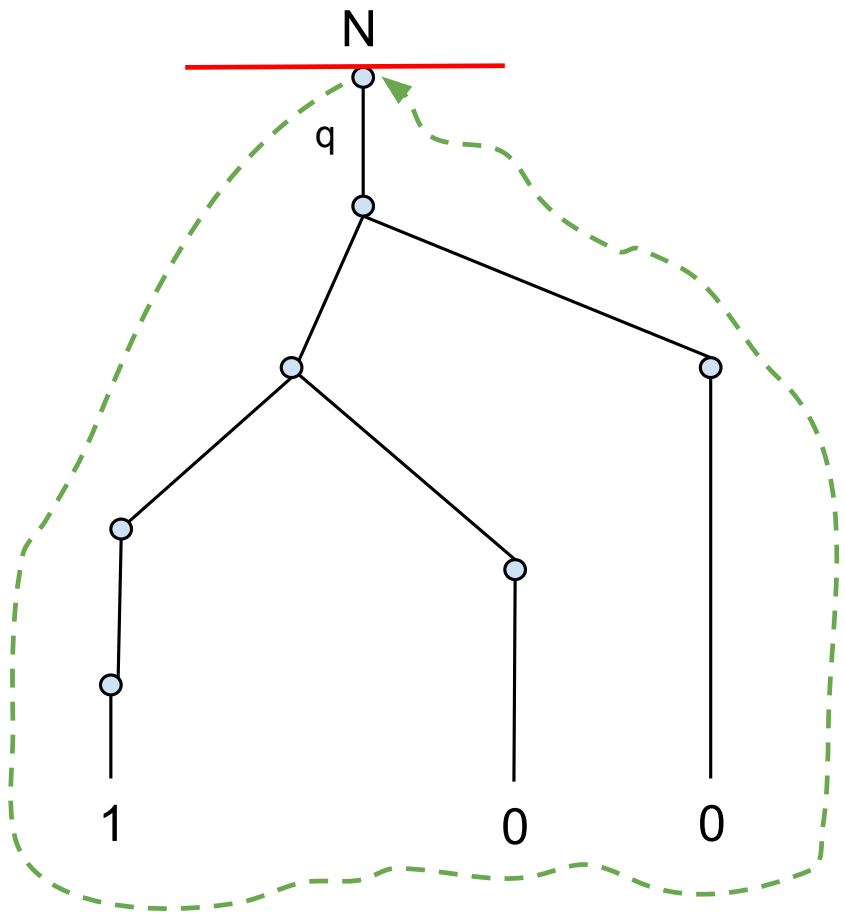
$$\texttt{data}\ \_|[\_]\overset{+/*}{\rightsquigarrow}[\_]\ \{Z : \texttt{N}\}\{t : \texttt{Tree}\ Z\}(b : \texttt{PosTree}\ t) : \forall\{X\ Y\ t_1\ t_2\} \to \texttt{PosTree}\ t_1 \times \texttt{Attr} \to \texttt{PosTree}\ t_2 \times \texttt{Attr} \to \texttt{Set}$$

$$q\ |[\ q\ ,\ v\ ]\overset{+}{\rightsquigarrow}[\ q\ ,\ s\ ]$$



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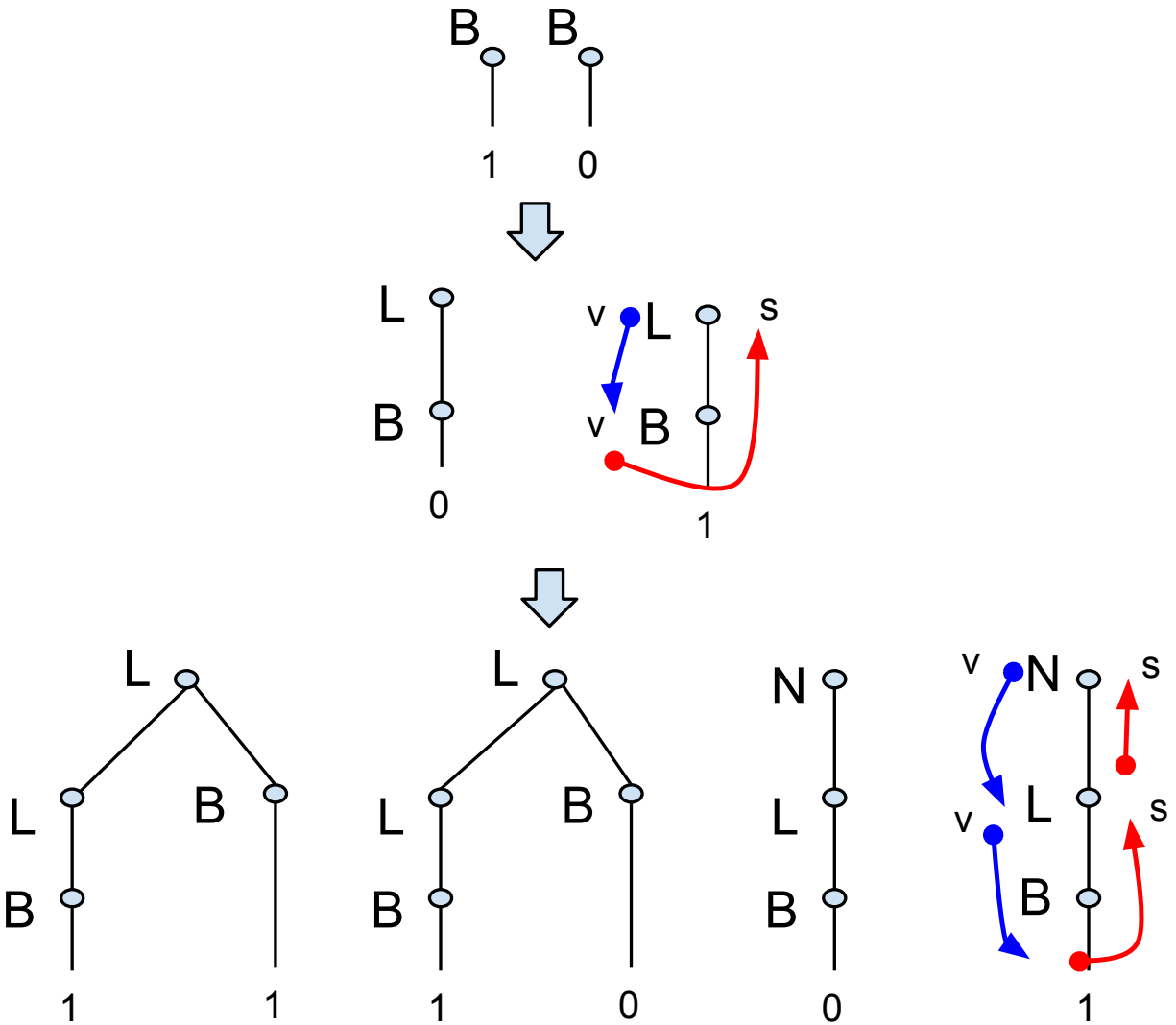
_≈_ : {X : N} → Tree X → Tree X → Set
t ≈ t' = dependencies t ≡ dependencies t'
```

Checking for circularity: Algorithm

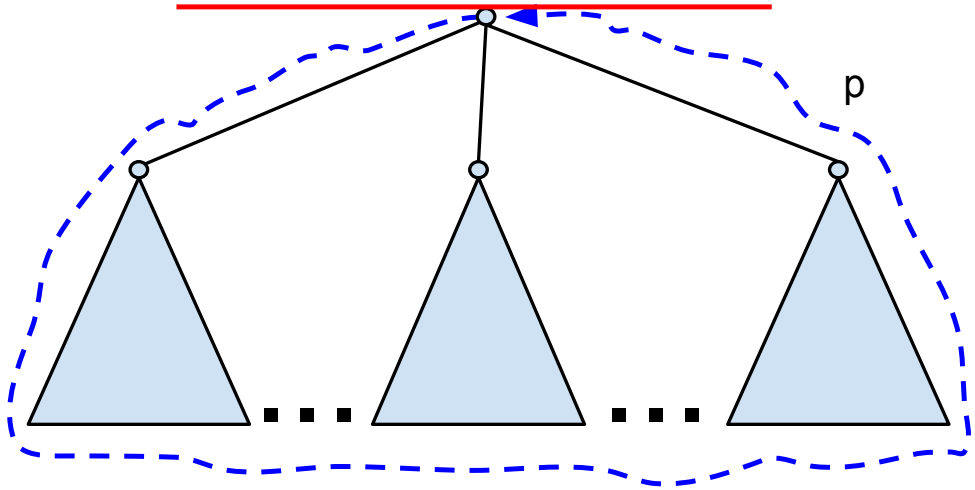
`treesToCheck : Rules` $\rightarrow$ `Forest`

1. Initialize accumulator with terminal rules
2. Build new trees from existing ones in accumulator
3. For each new tree `t`:
   1. Compute the `dependencies t`
   2. If there is no tree `t'` in accumulator such that `t` $\approx$ `t'` then add `t` to accumulator
4. If at least one tree was added then go to step 2, otherwise return accumulator.

Checking for circularity: Example



Cycle decomposition



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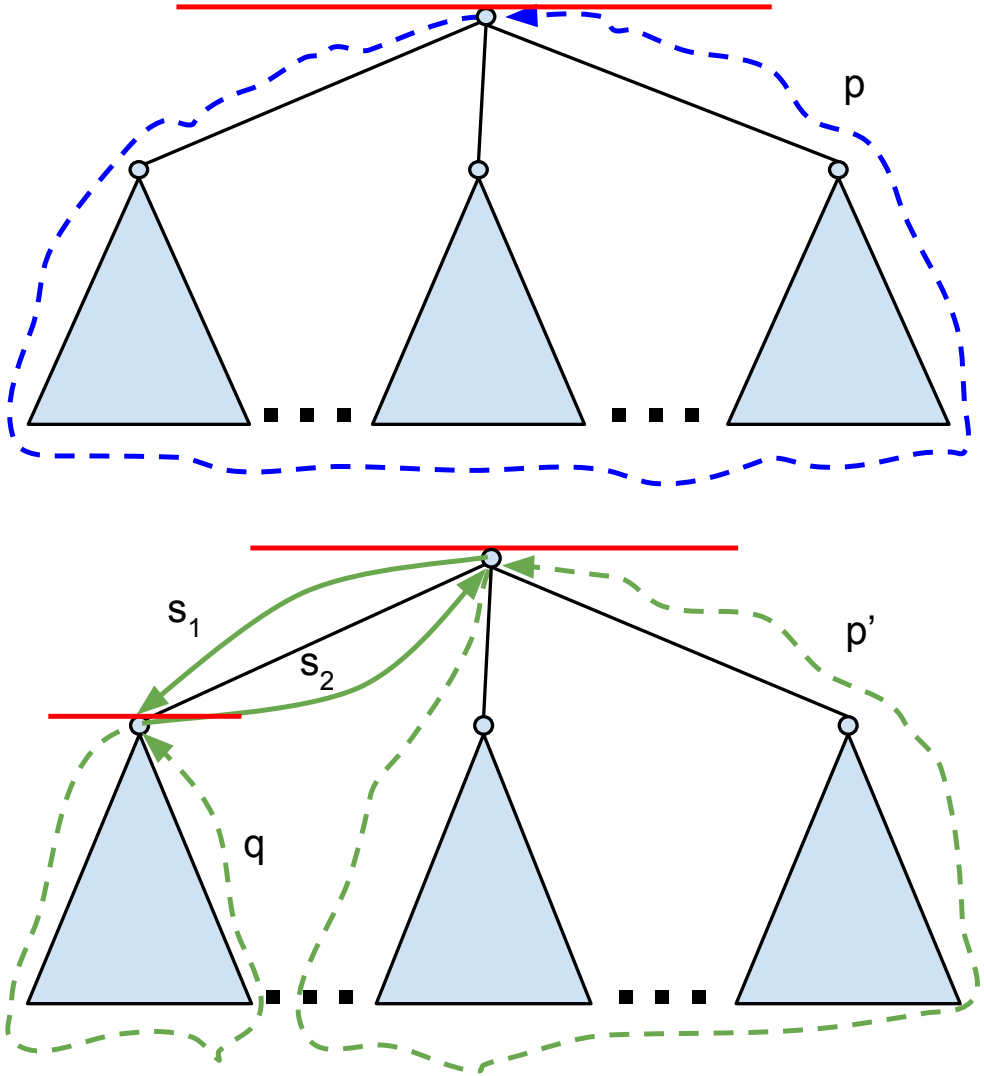
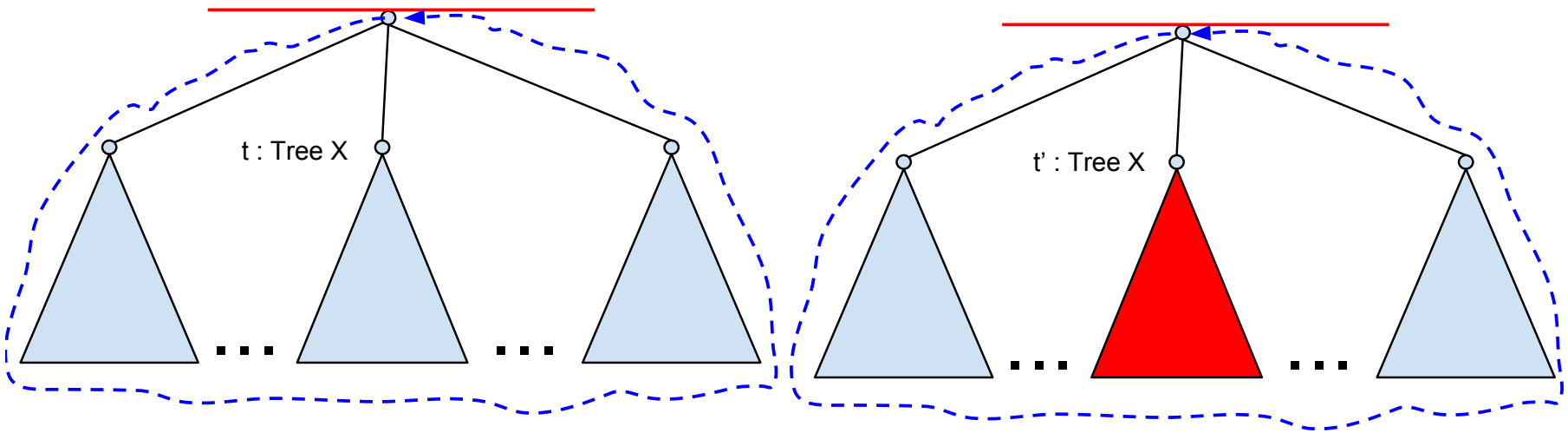


$$p \equiv s_1 \mathbin{+\!\!+} q \mathbin{+\!\!+} s_2 \mathbin{+\!\!+} p'$$

Substitution lemma

If `t` $\approx$ `t'` then
`node (Rule X (f1 ++ [ t ] ++ f2))` $\approx$ `node (Rule X (f1 ++ [ t' ] ++ f2))`



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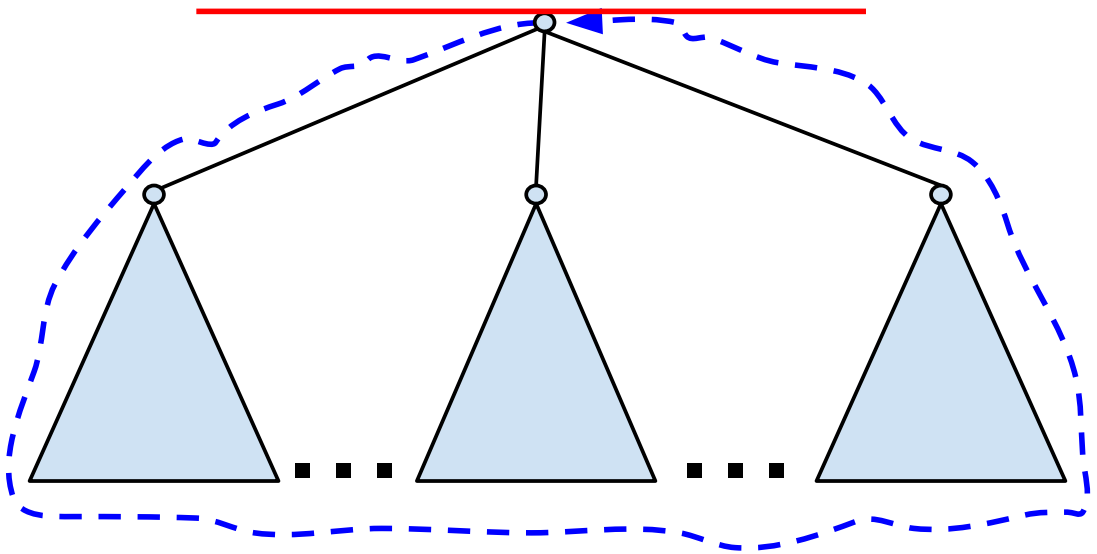

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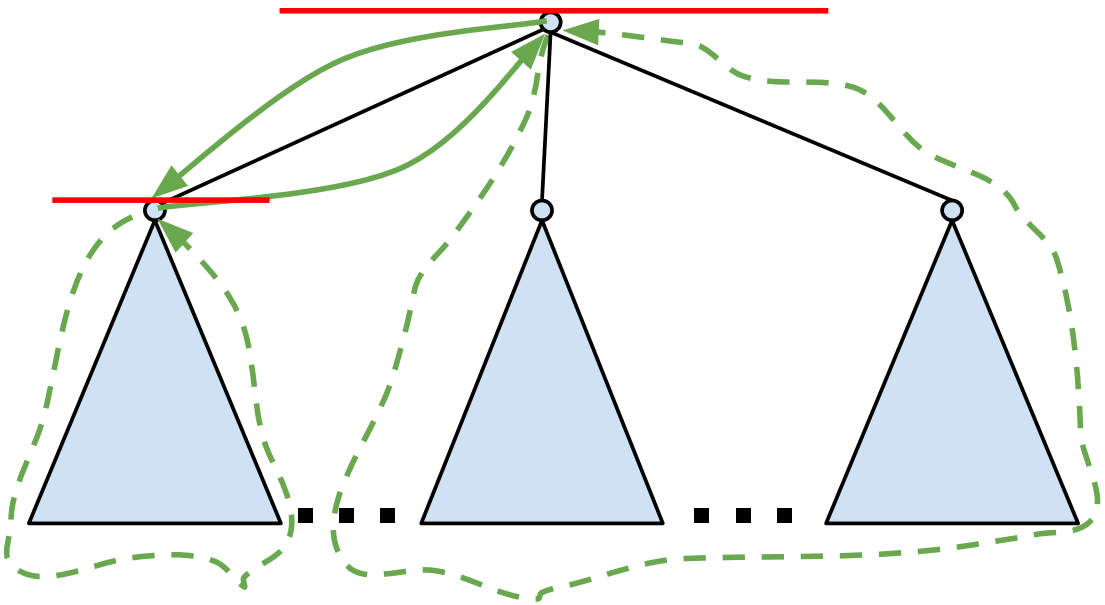

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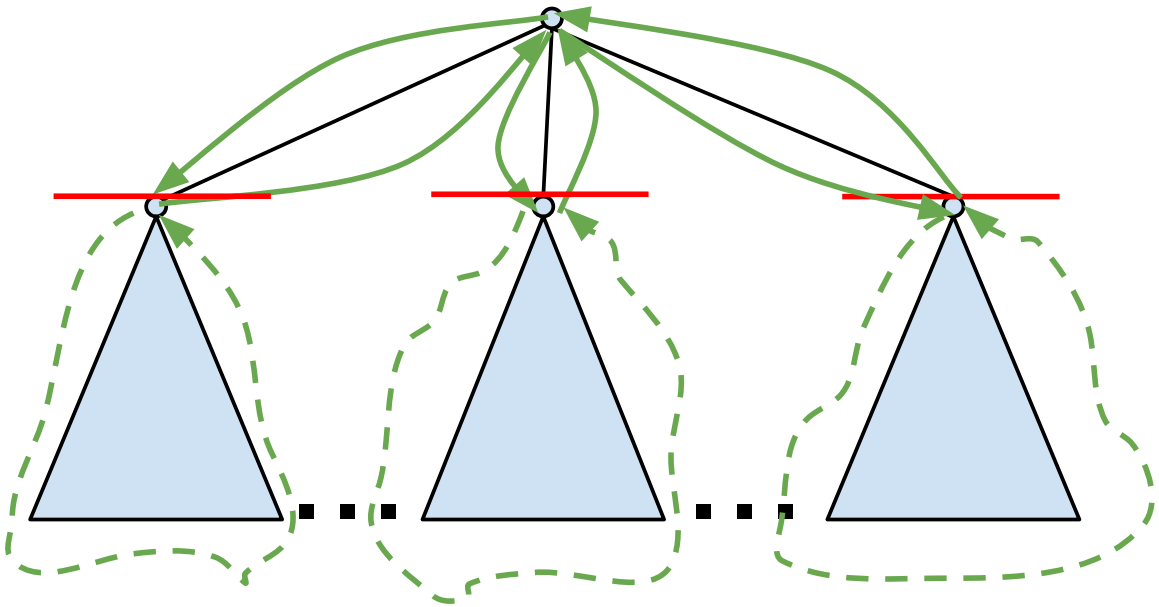

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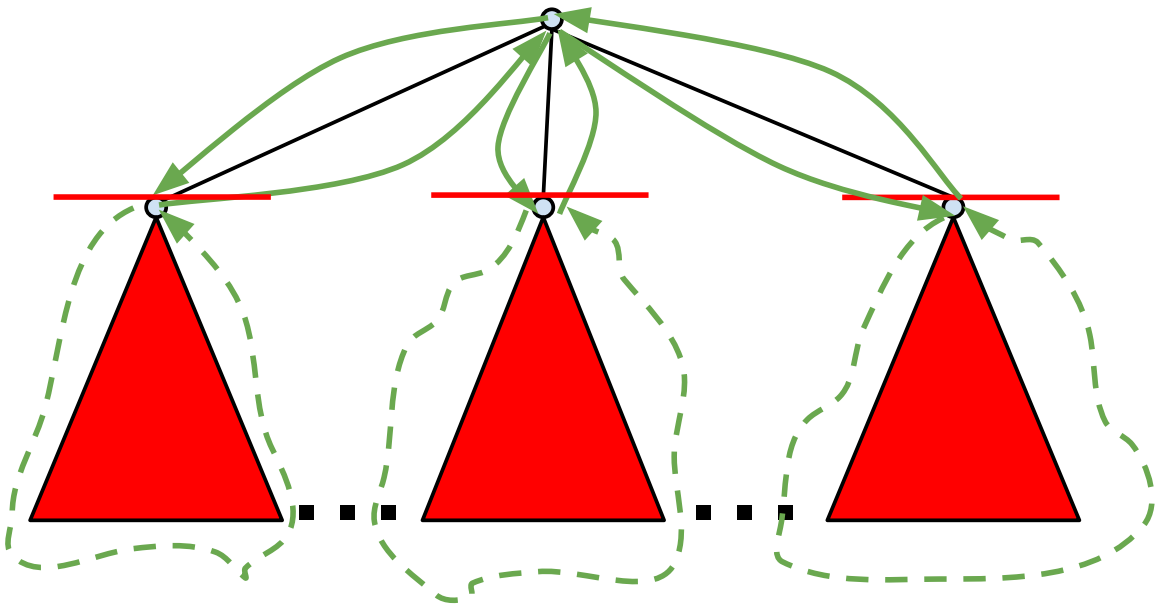

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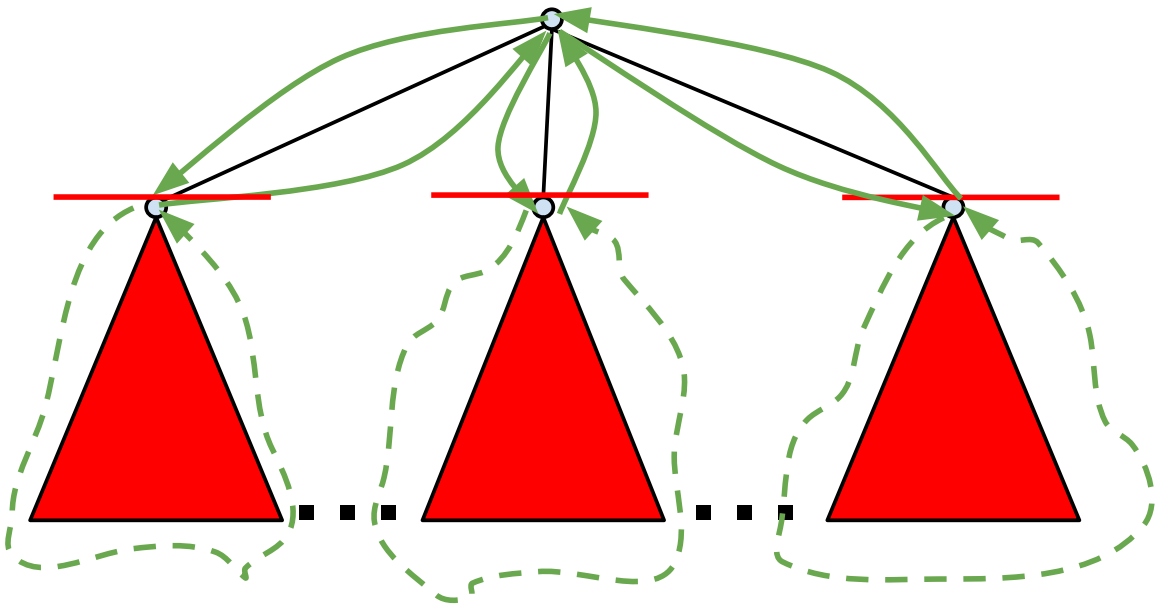

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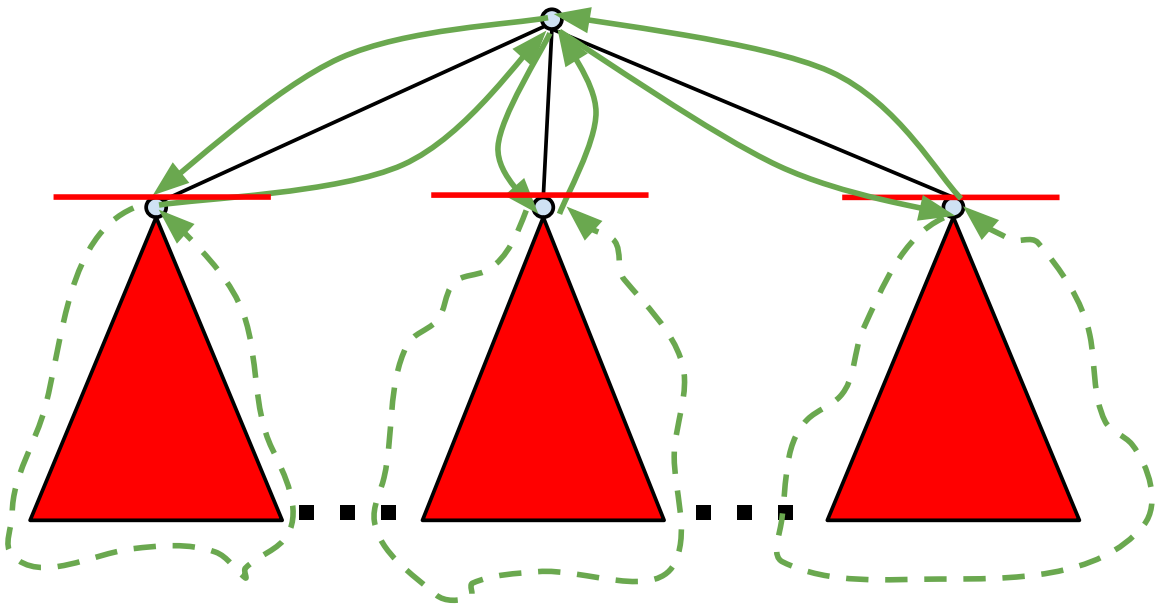
     → ∃ (t' : Tree X) , t ≈ t' × (X, t') ∈ treesToCheck Rs
```



```
circular? : Rules → Bool
circular? = ⋁ { a ≟ b | t ← treesToCheck Rs, (a, b) ← dependencies t }
```

Thank you for your attention!
Questions?