On the computational soundness of cryptographically masked flows

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Motivation

- Usual non-interference too strong for programs with encryption.
- Cryptographic security definitions
  - use complex domains,
  - are notationally heavy.
- The definitions for computational non-interference suffer from the same problems.
- Could we abstract from these definitions? Is there some formalism, where
  - the domain and the definition of non-interference were more "traditional",
  - NI for a program in this domain would mean computational NI for the "same" program in the real-world semantics?

Cryptographically masked flows

- Aslan Askarov, Daniel Hedin, Andrei Sabelfeld. Cryptographically-Masked Flows. SAS 2006.
- A proposal for the formalism that abstracts away complexity-theoretic details, but leaves (most of) everything else intact.
- Encryption is modeled non-deterministically.
- Possibilistic non-interference with extra leniency for encrypted values.

- Does NI in this model imply computational NI? Are cryptographically masked flows computationally sound?

The programming language

- In this talk: The WHILE-language with extra operations:
  - key generation, encryption, decryption
  - pairing, projection
- . . . and the usual:
  - Assigning expressions to variables
  - Sequential composition
  - If-then-else
  - While-loops
- In the [AHS06]-paper: more. . .
  - Parallel processes with global variables and message channels
  - Two encryption schemes (one for public values only)

Abstract semantics

Semantics

- Big-step SOS from a configuration to a set of final states.
  - Configuration — pair of the yet-to-be-executed program and the current state.
- The state consists of
  - The memory — mapping from variables to values;
  - The "key-stream" — the values of keys generated in the future.
- All operations, except encryption, are deterministic.

Encryption Systems

- Three algorithms:
  - $\mathcal{K}$ — key generation, zero arguments, probabilistic;
  - $\mathcal{E}$ — encryption, two arguments, probabilistic;
  - $\mathcal{D}$ — decryption, two arguments, deterministic.
- Correctness: $\mathcal{D}(k, \mathcal{E}(r; k, x)) = x$ for all
  - keys k that can be output by $\mathcal{K}$;
  - possible random coins r used by $\mathcal{E}$.
- The random coins used by $\mathcal{E}$ are called the *initial vector*.
- $\mathcal{D}$ may produce an error.

Semantics

- Big-step SOS from a configuration to a set of final states.
- The state consists of
  - The memory — mapping from variables to values;
  - The "key-stream" — the values of keys generated (by $\mathcal{K}$) in the future.
- All operations, except encryption, are deterministic.
- Encryption models the randomized encryption algorithms of the real world:
  - To encrypt x with the key k, choose an *initial vector* r and compute $\mathcal{E}(r; k, x)$.
  - In reality, r is chosen probabilistically, here it is modeled by non-deterministic choice.

Low-equivalence of memories

- Let the variables be partitioned to $\mathbf{Var}_{\mathrm{H}}$ and $\mathbf{Var}_{\mathrm{L}}$.
- Let the values be tagged with their types — key, encryption, pair, other (integer).
- $n \sim_{\mathrm{L}} n$;
- $k \sim_{\mathrm{L}} k$;
- $x_1 \sim_{\mathrm{L}} y_1 \wedge x_2 \sim_{\mathrm{L}} y_2 \Rightarrow (x_1, x_2) \sim_{\mathrm{L}} (y_1, y_2)$;
- $\mathcal{E}(r; k_1, x_1) \sim_{\mathrm{L}} \mathcal{E}(r; k_2, x_2)$ for all x_1, x_2, k_1, k_2.
- $S_1 \sim_{\mathrm{L}} S_2$ if $S_1(x) \sim_{\mathrm{L}} S_2(x)$ for all $x \in \mathbf{Var}_{\mathrm{L}}$.

Possibilistic non-interference

Program P is non-interfering if

- for all states S_1, S_2 and keystreams G_1, G_2, such that $S_1 \sim_{\mathrm{L}} S_2$
- let $\mathcal{S}_i = \{S' \,|\, (S_i, G_i) \longrightarrow (S', G')\}$ for $i \in \{1, 2\}$, then
- for all $S'_1 \in \mathcal{S}_1$
- there must exist $S'_2 \in \mathcal{S}_2$
- such that $S'_1 \sim_{\mathrm{L}} S'_2$.

(and vice versa)

Equivalently: Given a state S and keystream G, let

$$\mathcal{S} = \left\{ \lambda v. \begin{cases} \text{coins}(S'(v)), & S'(v) \text{ is ciphertext} \\ S'(v), & \text{otherwise} \end{cases} \,\middle|\, (S, G) \longrightarrow S' \right\}$$

Then $\mathcal{S}$ may depend only on the values of low-variables in S.

Concrete semantics

"Real-world" semantics

- Big step SOS — maps an initial configuration to a probability distribution over final states.
  - Let us not consider non-termination.
  - And assume that the program terminates in a reasonable number of steps.
- Initial state is distributed according to some D.
- The program P is non-interferent if no algorithm $\mathcal{A}$ using a reasonable amount of resources can guess b from

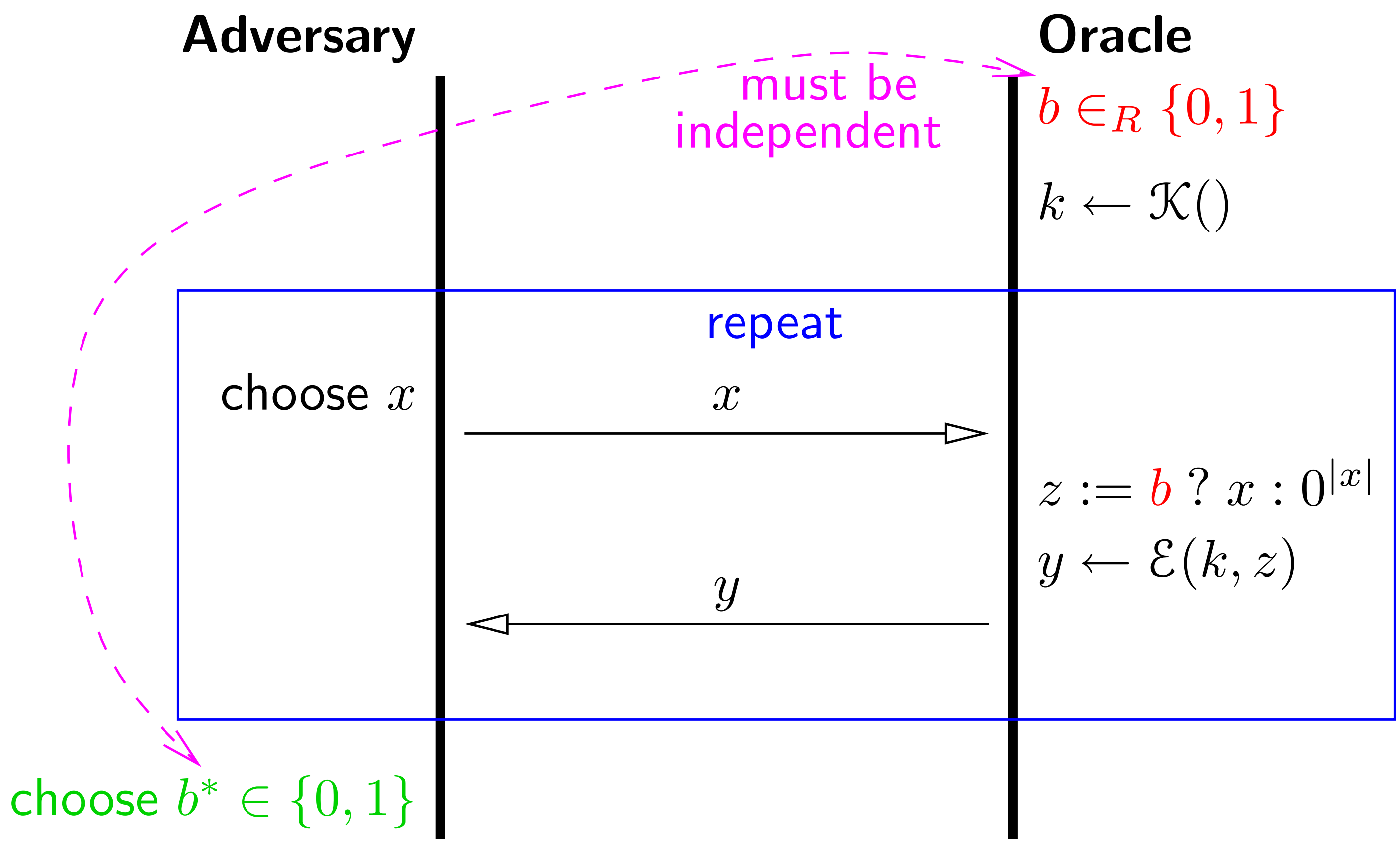
$$\begin{array}{l} b \leftarrow_R \{0,1\},\ S_0, S_1 \leftarrow D \\ S' \leftarrow [\![P]\!](S_b) \\ \text{give } (S_0|_{\mathbf{Var}_{\mathrm{H}}}, S'|_{\mathbf{Var}_{\mathrm{L}}}) \text{ to } \mathcal{A} \end{array}$$

Soundness theorem

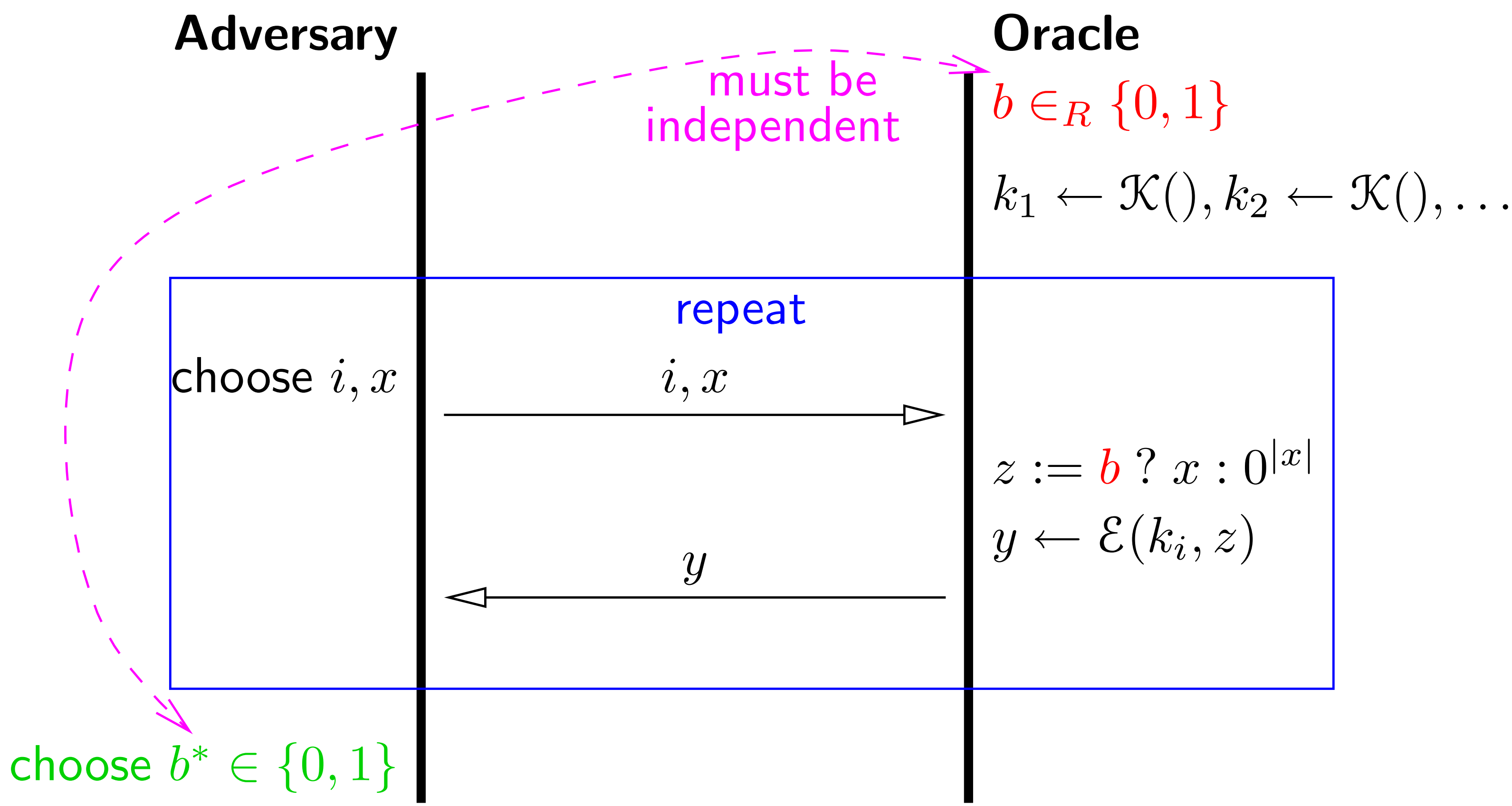
- If the program P satisfies the following conditions:
  - ...
- and the encryption system satisfies the following conditions
  - IND-KDM-CPA- and INT-KDM-PTXT-security
- and P satisfies possibilistic non-interference
- then P satisfies computational non-interference.

- The conditions put on P should be verifiable in the possibilistic model.
  - Otherwise we lose the modularity of the approach.

IND-CPA


Adversary
Oracle
must be independent
$b \in_R \{0,1\}$
$k \leftarrow \mathcal{K}()$
repeat
choose x
x
$z := b \;?\; x : 0^{|x|}$
$y \leftarrow \mathcal{E}(k,z)$
y
choose $b^* \in \{0,1\}$

IND-CPA with several keys



- Equivalent to the previous one.

IND-KDM-CPA

Adversary | **Oracle**

$b \in_R \{0,1\}$

$k_1 \leftarrow \mathcal{K}(), k_2 \leftarrow \mathcal{K}(), \ldots$

must be independent

repeat

choose $i, e(x_1, x_2, \ldots)$

i, e

$w := e(k_1, k_2, \ldots)$

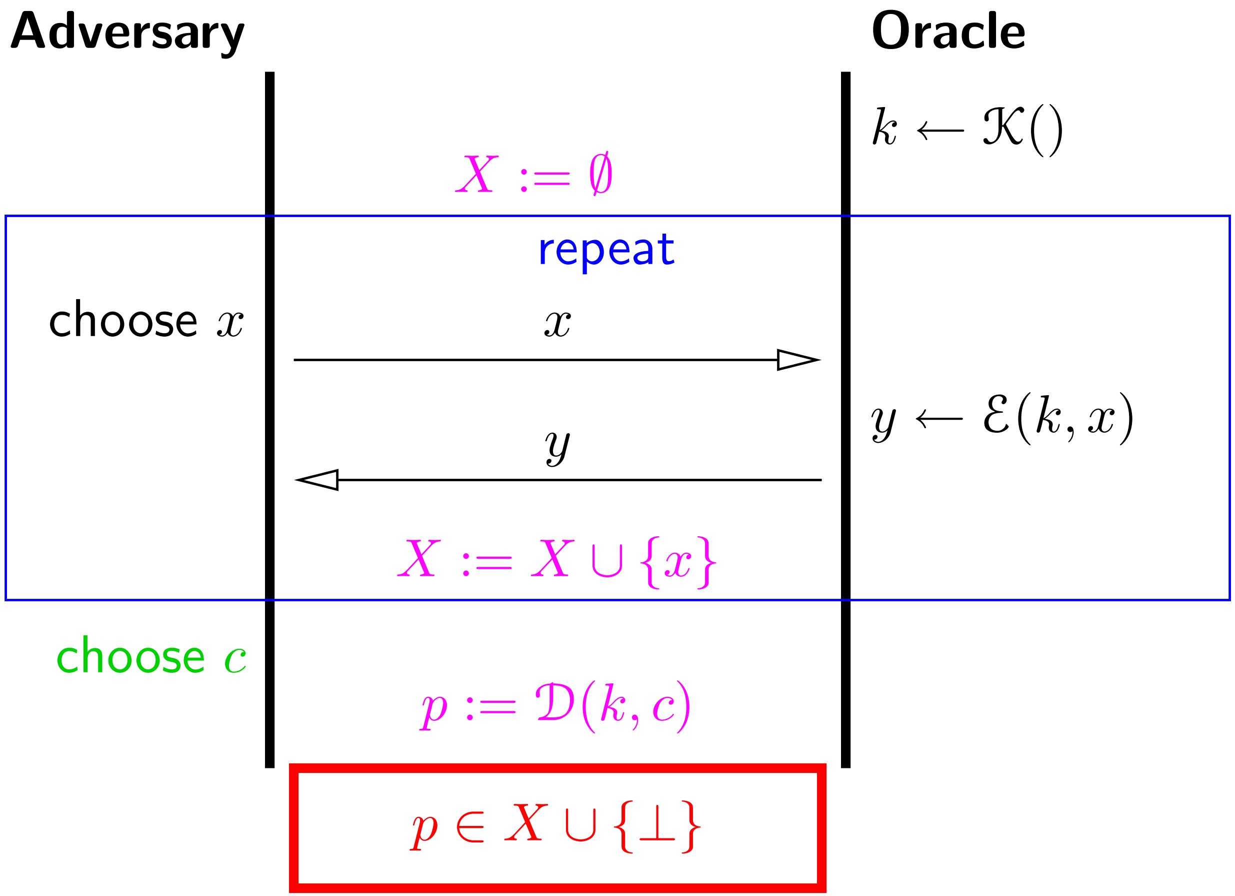
$z := b \; ? \; w : 0^{|w|}$

$y \leftarrow \mathcal{E}(k_i, z)$

y

choose $b^* \in \{0,1\}$

INT-PTXT



- Similarly define INT-PTXT with several keys and INT-KDM-PTXT.

Condition: ciphertexts only from $\mathcal{E}$

- $\sim_{\mathrm{L}}$'s relaxed treatment of ciphertexts must be restricted to values produced by the encryption operation.
- Otherwise, consider the following program:

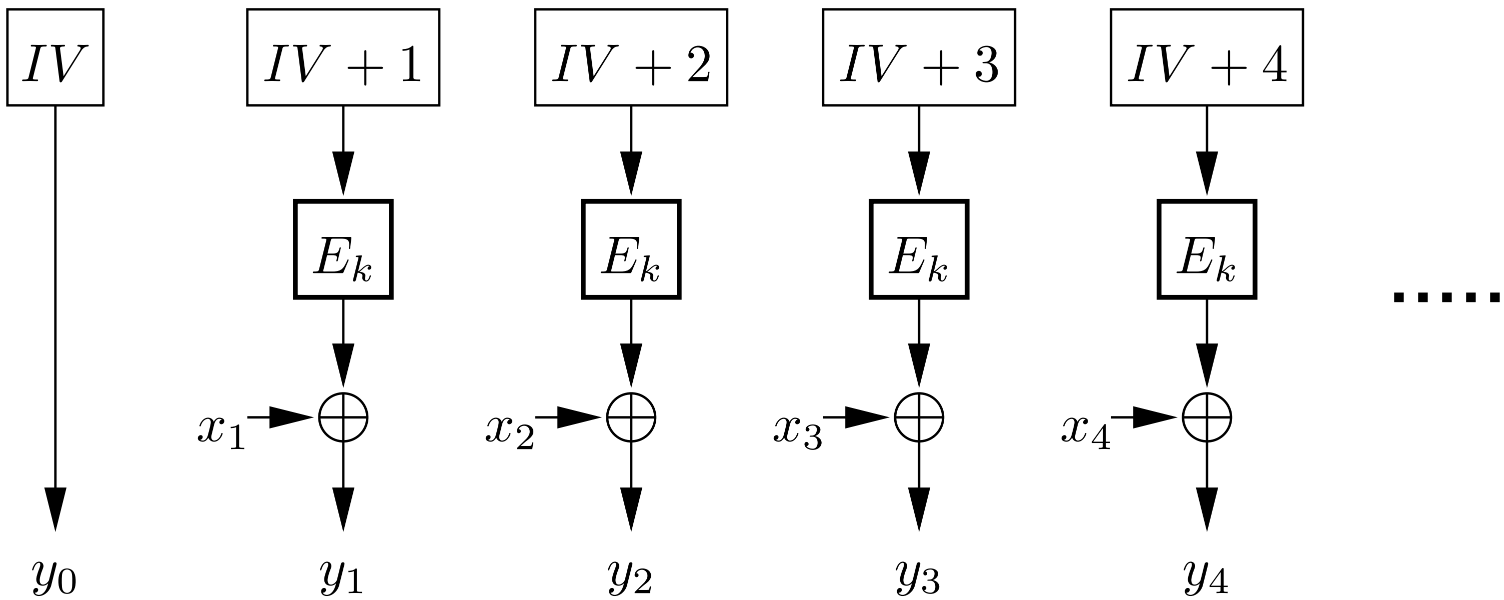
$$k := \mathsf{newkey}; p_1 := \mathsf{enc}(k, s)$$
$$r := \mathsf{getIV}(p_1); p_2 := \widetilde{\mathsf{enc}}(r + 1; k, s)$$

- Initial state $(\{s \mapsto v_s\}, v_k :: G)$ is mapped to

$$\left\{ \{p_1 \mapsto \mathcal{E}(v_r; v_k, v_s), p_2 \mapsto \mathcal{E}(v_r + 1; v_k, v_s)\} \middle| v_r \in \mathbf{Coins} \right\}$$

that does not depend (for $\sim_{\mathrm{L}}$) on initial secrets.

Counter mode of using a block cipher



- A good encryption system (IND-CPA).
- If we used it on the previous slide, then we could learn $v_{s1} \oplus v_{s2}$, $v_{s2} \oplus v_{s3}$, $v_{s3} \oplus v_{s4}$, ...

Condition: keys used only at $\mathcal{E}$ and $\mathcal{D}$...

- ... and vice versa.
- Consider the program

$$k_1 := \mathsf{newkey}; \mathbf{if}\ \mathsf{B}(k_1)\ \mathbf{then}\ k_2 := k_1\ \mathbf{else}\ k_2 := \mathsf{newkey}\ \mathbf{fi}; \ldots$$

- Afterwards, k_2 is not distributed as coming from $\mathcal{K}$.

Enforcing those conditions

- Give types to variables: the types τ are

$$\tau ::= int \mid key \mid enc(\tau) \mid (\tau, \tau)$$

- We may want to compute with ciphertexts, hence we subtype $enc(\tau) \leq int$.
- Types of operations:
  - arithmetic operations: $int^k \rightarrow int$;
  - pairing: $\tau_1 \times \tau_2 \rightarrow (\tau_1, \tau_2)$; i-th projection: $(\tau_1, \tau_2) \rightarrow \tau_i$;
  - key generation: $\mathbf{1} \rightarrow key$;
  - encryption: $key \times \tau \rightarrow enc(\tau)$; decryption: $key \times enc(\tau) \rightarrow \tau$;
  - guards: int.
- [AHS06] already has such a type system.

Part of the proof: Removing decryptions

- Change the real-world program:
  - Give names to keys: replace each $k :=$ newkey with

$$k := \text{newkey}; k_{\text{name}} := c; c := c + 1$$

  - for each ciphertext record the key name and the plaintext in the auxiliary variables. Replace $y := \mathcal{E}(k, x)$ with

$$y := \mathcal{E}(k, x); y_{\text{keyname}} := k_{\text{name}}; y_{\text{ptext}} := x$$

  - Replace the statements $x := \mathcal{D}(k, y)$ with

$$\textbf{if } k_{\text{name}} = y_{\text{keyname}} \textbf{ then } x := y_{\text{ptext}} \textbf{ else } x := \perp \textbf{ fi}$$

- The low-visible semantics does not change.

Encryption $\rightarrow$ random number gen.-tion

- Apply the definition of IND-KDM-CPA to the real-world program:
  - Replace each $\mathcal{E}(k, y)$ with $\mathcal{E}(k_0, 0)$.
- $\mathcal{E}(k_0, 0)$ generates random numbers according to a certain distribution.
- In the possibilistic NI, we also treat encryption as random number generation.
  - As only the initial vector matters.

Possib. secrecy $\not\Rightarrow$ probab. secrecy

- Let h be a number from 1 to 100. Consider the following program

$$\mathbf{if}\ \mathsf{rnd}(\{0,1\}) = 1\ \mathbf{then}\ l := h\ \mathbf{else}\ l := \mathsf{rnd}(\{1,\ldots,100\})$$

- The possible values of l do not depend on h.
- But their distribution depends on h.
- We can come up with similiar examples in our language.
  - Using $\mathcal{E}$ in place of rnd.
- Hence using ciphertexts in computations is questionable as well.
- Remove the subtyping $enc(\tau) \leq int$.

The conditions for the program

- The variables are typed, as specified before.

$$\tau ::= int \mid key \mid enc(\tau) \mid (\tau, \tau)$$

  (no subtyping)
- The operations respect those types.
- Failures to decrypt are visible in the possibilistic semantics.

On plaintext integrity

Consider the following program:

$$
\begin{array}{l}
k := \mathcal{K}();\ k' := \mathcal{K}();\ x := \mathcal{E}(k, C);\ y := \mathcal{D}(k', x); \\
\mathit{if}\ \ y = \perp\ \mathit{then}\ l := h\ \mathit{else}\ l := 1 - h
\end{array}
$$

- There may be some (negligible) chance that the decryption succeeds.
- Thus, in the abstract semantics, $else$-branch can be taken.
  - In the abstract semantics, this program is secure.
- In concrete semantics, $l = h$ with overwhelming probability.

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- There may be some (negligible) chance that the decryption succeeds.

- Thus, in the abstract semantics, $else$-branch can be taken.
  - In the abstract semantics, this program is secure.
- In concrete semantics, $l = h$ with overwhelming probability.
- We exclude this case by modifying the abstract semantics.
  - Do not allow two generated keys to be the same.
  - Record the keys for generated ciphertexts. Do not allow decryption with the wrong key.

Theorem

- Conditions on the program:
  - It types dynamically according to the given type system.
    - Current types of variables are a part of the state.
  - Uses only initial values of type int.
  - Has possibilistic non-interference.
- The encryption scheme must be IND-KDM-CPA- and INT-KDM-PTXT-secure.

Then the program has probabilistic non-interference.

Conclusions

- Cryptographically masked flows still put serious restrictions on the manipulation of the results of cryptographic operations.
- The restrictions are similar to the Dolev-Yao model:
  - using keys only as keys or in operations where the value remains opaque (pairing and encryption);
  - ciphertexts may only be decrypted on used in operations where the value remains opaque.
- In fact, we can formulate an equivalent model with symbolic encryptions, and get rid of the non-determinism.
- We'd like to have a model
  - without probabilities;
  - where ciphertexts (and keys) may be used as values in (m)any computations.

But this may be impossible...