Side Effect Monad, its Equational Theory and Applications

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Seminar, 2005

Shkaravska Side-effect Monad

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Motivation

- Adding Imperative Features to Functional Programs
- Previous Works

Our Results

Categorical Semantics Of View-Update Problem

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Adding Imperative Features to Functional Programs Previous Works

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Categorical Semantics Of View-Update Problem

Adding Imperative Features to Functional Programs Previous Works

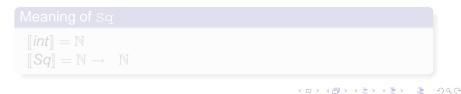
Pure Languages

Pure functional languages do not subsume:

- variable assignments x := 2,
- field updates *x*.tail := another_list

The example stolen from G. Plotkin's talk

function *Sq(x : int) : int* return *X * X* end



Adding Imperative Features to Functional Programs Previous Works

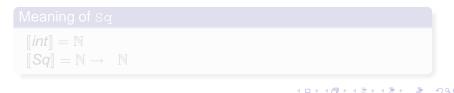
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Meaning of Sq
$$\llbracket int \rrbracket = \mathbb{N}$$
 $\llbracket Sq \rrbracket = \mathbb{N} \rightarrow \mathbb{N}$

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Pure Languages

Absence of side-effects: Advantages

Convenient reasoning about pure FL, especially CBV.

Example: heap-aware type systems. We use functional structures to verify heap consumption by a bytecode.

Absence of side-effects: Disadvantages

- One often needs to update fields ...
- CBV: unefficient usage of heap space

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Impure Language = Pure Language + Side Effects

Another example stolen from G. Plotkin's talk

function	Sq(x:int):int
<i>y</i> := 3	
return	X * X
end	

Meaning of Sq II

$$\begin{bmatrix} Sq \end{bmatrix} = \mathbb{N} \times S \to \mathbb{N} \times S$$

where $S = \mathbb{N}^{Loc}$

Equivalently $[Sq] = \mathbb{N} \to (\mathbb{N} \times S)^S$

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Impure Languages for Databases?

Intuition behind this Idea

The current content of the data Base is a state.

Programming with D-Bases

is a functional programming with side effects: *select* and *update* operations and functions on data.

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E. Moggi: Programs with Monads

Kleisli Kategory

 $\mathsf{Sq}:\mathbb{N}\to\mathbb{N}$

 $Sq: \mathbb{N} \to T_{state}(\mathbb{N}),$ with $T_{state}(\mathbb{N}) = (\mathbb{N} \times S)$ $\text{Div}: \mathbb{N} \to \mathbb{N}$ becomes $\text{Div}: \mathbb{N} \to T_{\text{Exception}}(\mathbb{N})$

 $P: A \to B$ becomes $P: A \to T(B)$

Adding Imperative Features to Functional Programs Previous Works

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Div:
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Previous Works

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 $P: A \rightarrow B$ becomes $P: A \rightarrow T(B)$

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Composition for Programs with Monads

Composition for "pure" programs

$P1: A \rightarrow B$	$P2: B \rightarrow C$
compose to	
$P1; P2 = P2 \circ P1:$	$A \rightarrow C$

Composition for monadic programs

```
Monadic programs = Kleisli arrows.

P1: A \rightarrow T(B) P2: B \rightarrow T(C)

compose to

P1; P2^* = P2 \bullet P1: A \rightarrow T(C)

Additional machinery

\_*: (f: A \rightarrow T(B)) \mapsto (f^*: T(A) \rightarrow T(B))
```

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Composition for Programs with Monads

Associativity

$P3 \bullet (P2 \bullet P1) = (P3 \bullet P2) \bullet P1$

means

$$(P1; P2^*); P3^* = P1; (P2; P3^*)^*$$

$$(f^*; g^*) = (f; g^*)^*$$

P1; (P2; P3*)* = P1; (P2*; P3*) = (P1; P2*); P3*

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Motivation	
Our Results	
Conclusions	

Identities

Why do we need the following map?

$$\eta_{\mathcal{A}}: \mathcal{A} \to \mathcal{T}(\mathcal{A})$$

(BTW, an element of T(A) is called a *computation*)

As a respectable programming language our "pure", original, one, has a program-which-do-nothing:

$$P: A \to B$$

$$P \circ \mathbf{id}_A = P \quad \text{that is} \quad \mathbf{id}_A; P = P$$

$$\mathbf{id}_B \circ P = P \quad \text{that is} \quad P; \mathbf{id}_B = P$$

What should be identities for the monadic langauge? $A \rightsquigarrow A$ is $A \rightarrow T(A)$

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Adding Imperative Features to Functional Programs Previous Works

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Kleisli Triple

Definition

$$T(A) = (A \times S)^{S}$$

$$\eta_{A} : a \mapsto \lambda s : S. (a, s)$$

$$(f : A \to T(B)) \mapsto (f^{*} : T(A) \to T(B))$$

s.t. $f^{*}(c) == \lambda s : S. let (a, s') = c(s) in f(a)(s')$

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Strength

$$t_{A, B}: A \times T(B) \rightarrow T(A \times B)$$

Compare a "simple" let and a let with nonlinear usage of variables.

$$T(A) = (A \times S)^{S}$$

$$t(a, c) == \lambda s: S. let (b, s') = c(s) in ((a, b), s')$$

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Axiomatics

Side-Effects

 $S = V^{Loc}$

$$\begin{aligned} & \text{sel}(\textit{upd}(a, \textit{loc}, \textit{v}), \textit{loc}) = \textit{v} \\ & \textit{upd}(a, \textit{loc}, \textit{sel}(a, \textit{loc})) = a \\ & \textit{upd}(\textit{upd}(a, \textit{loc}, \textit{v}), \textit{loc}, \textit{v}') = \textit{upd}(a, \textit{loc}, \textit{v}') \\ & \textit{upd}(\textit{upd}(a, \textit{loc}, \textit{v}), \textit{loc}', \textit{v}') = \textit{upd}(\textit{upd}(a, \textit{loc}', \textit{v}'), \textit{loc}, \textit{v}), \\ & \textit{where loc} \neq \textit{loc}' \end{aligned}$$

Positive Subtyping

```
get(put(c, a)) = a

put(c, get(c)) = c

put(put(c, a), a') = put(c, a')
```

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Adding Imperative Features to Functional Programs Previous Works

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View-Update Problem

Views of a database, concrete or abstract, are its **states**. Sets of views, *C* and *A*, determine the corresponding **state monads**, $(C \times (-))^C$ and $(A \times (-))^A$. A total lens *I* is a pair of maps, *get*,

$$I \nearrow : C \to A$$

and putback,

$$\searrow: \mathbf{C} \times \mathbf{A} \to \mathbf{C}.$$

A lens is called *very well behaved* if its components subject to three axioms:

$$\begin{split} I\searrow(I\nearrow c,\ c) &= c & (\text{GetPut})\\ I\nearrow(I\searrow (a,\ c)) &= a & (\text{PutGet})\\ I\searrow(a',\ I\searrow (a,\ c)) &= I\searrow (a',\ c) & (\text{PutPut}) \end{split}$$

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Categorical Semantics Of View-Update Problem

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Outline



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Categorical Semantics Of View-Update Problem

Categorical Semantics Of View-Update Problem

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Categorical Semantics Of View-Update Problem

Theorem. Given a very well behaved lens *I*, one can construct a functor from $KI(T_A)$ onto $KI(T_C)$.



- Programming over data bases may be considered as functional programming with side effects
- A very-well behavied lens defines a map of Kleisli categories

Future Work

- To which extend aur assumption is correct? What can it bring to data base world?
- Very-well behavied lens defines a monad morphism

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